6 Easy-to-use (ETU) Tools
for
Uncertainty Quantification (UQ)
with
Example Applications to
Engineering Research & Practice (*)

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Gaithersburg, MD 20899
http://www.nist.gov/itl/math/Jeffrey-t-fong.cfm

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Outline of Talk (1 + 2 + 3 + 4 + 8 = 60 slides)

1. Why is UQ important in Engineering ?
2. Example of an *Easy-to-use* UQ Tool for Engineers.
3. Six *Easy-to-use* (*ETU*) Tools of Engineering UQ .
4. Tool-2, 3, 4, 5. UQ for Brain Metrology Research.
5. **Tool-3, & 5. UQ for Flaw Detection and Sizing.**
6. **Tool-1 & 2. UQ for Design of an Aircraft Window.**
7. **Tool-6. UQ for Maintenance Decision Making.**
8. Concluding Remarks.

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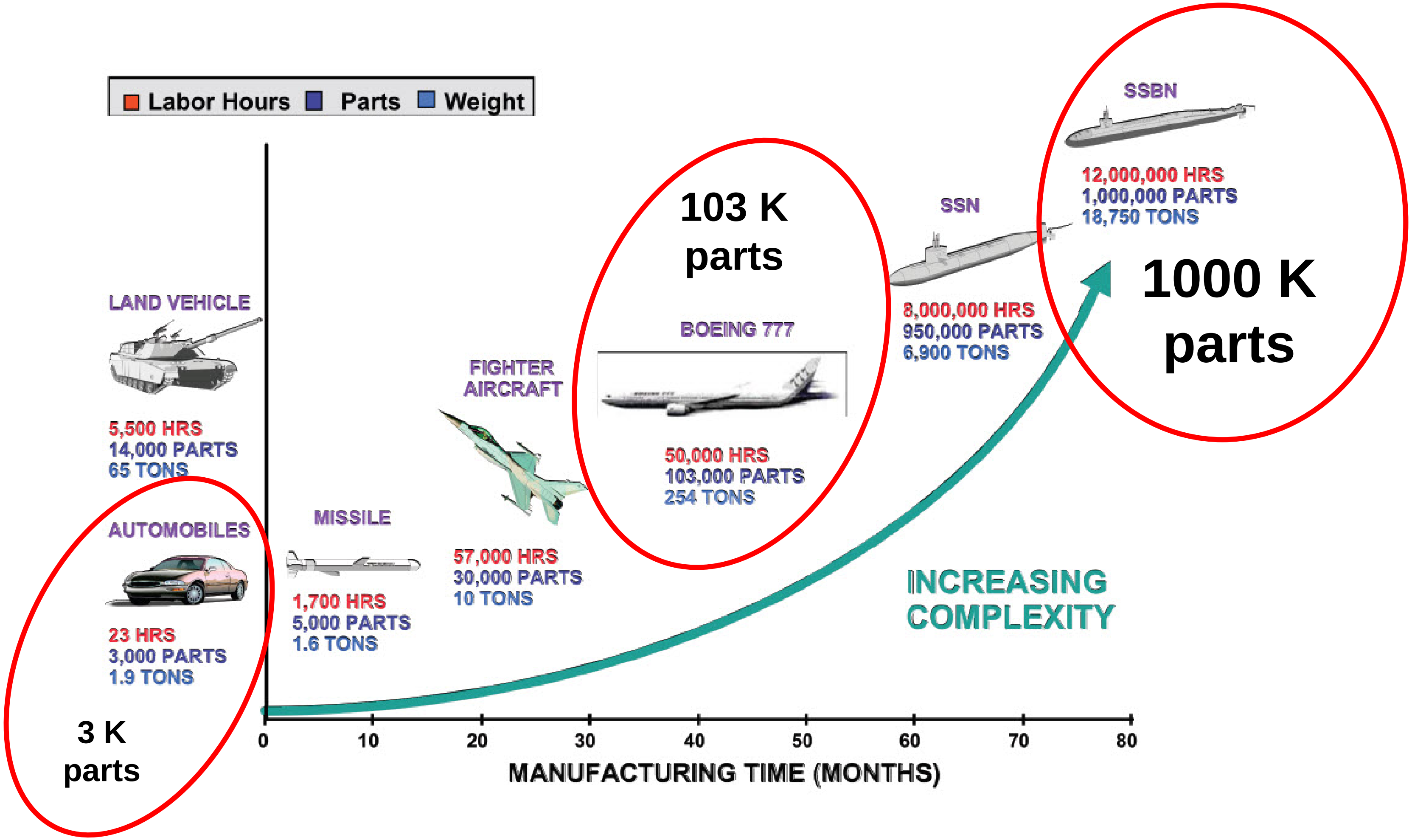
Email: fong@nist.gov

1. Why is UQ important in Engineering ?

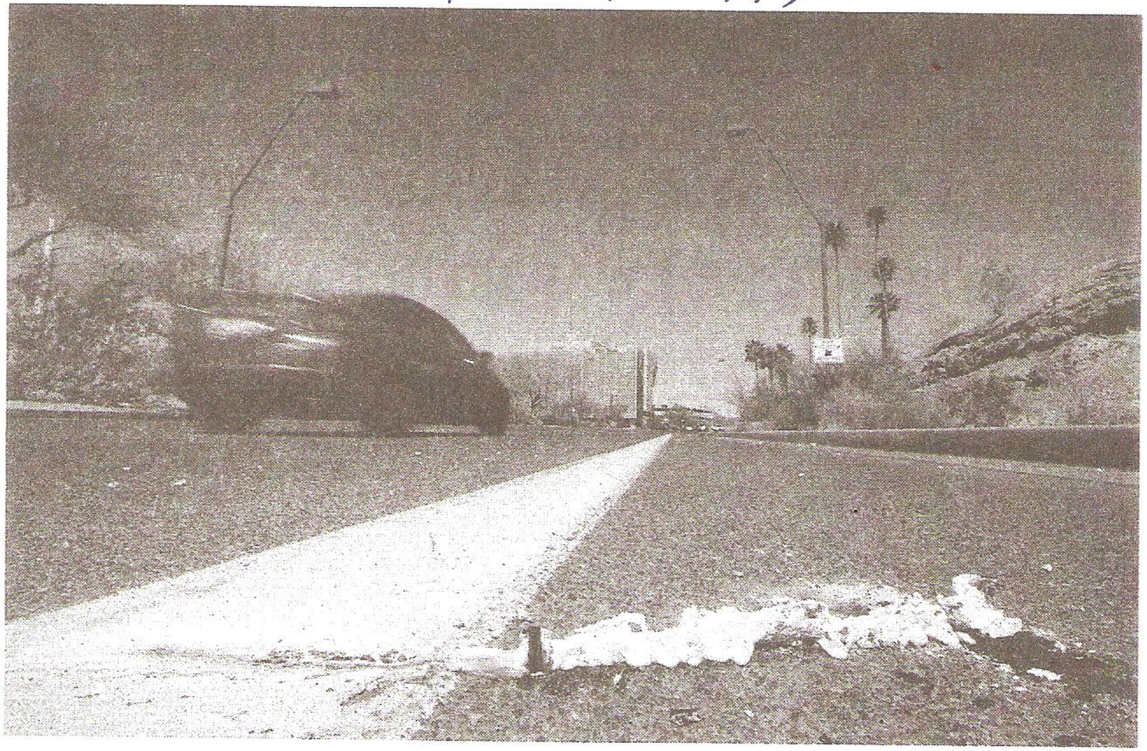


Courtesy of General Dynamics / Electric Boat Corporation

1. Why is UQ important in Engineering ?

Question-1: When artificial intelligence (AI) makes a lethel mistake, how do we assess blame?

Washington Post, Sunday, Mar. 25, 2018, page B5



CHRIS CARLSON/ASSOCIATED PRESS

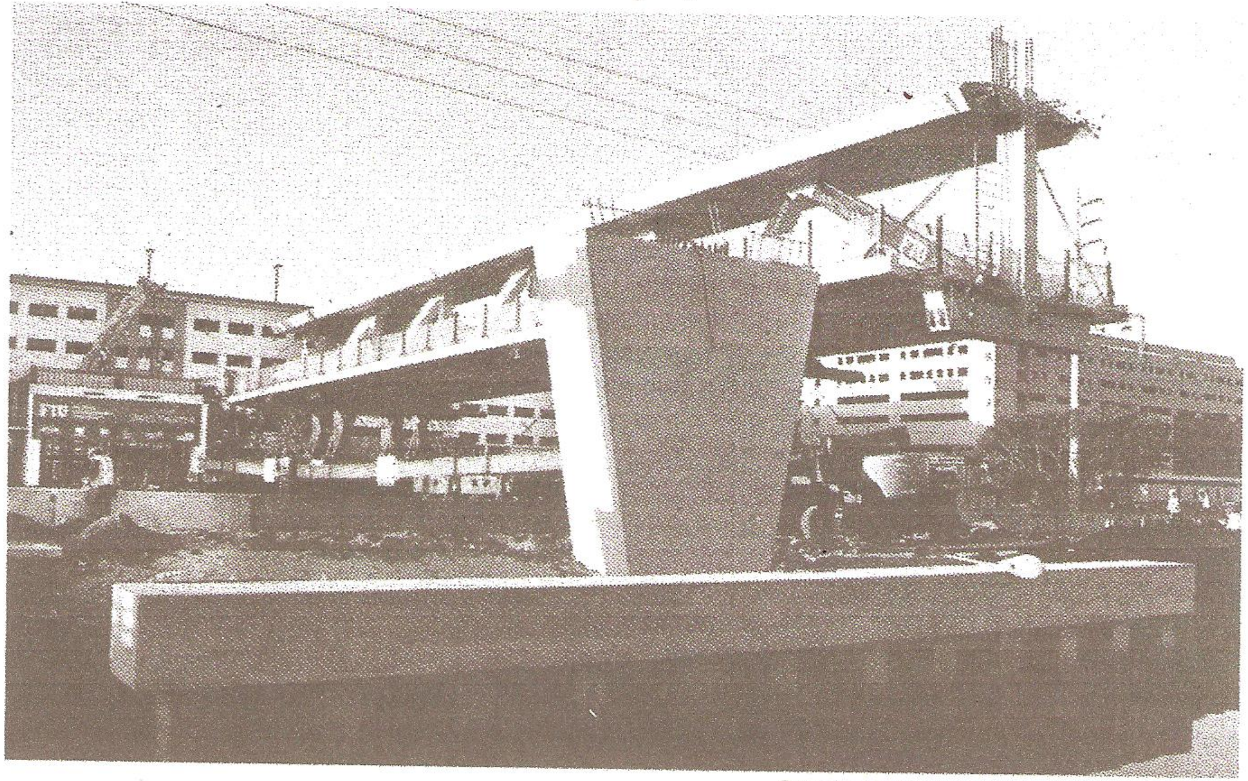
The location in Tempe, Ariz., where a self-driving Uber vehicle struck a pedestrian last weekend. Artificial intelligence systems can't always explain what they were "thinking" when such accidents occur.

Sunday
Mar. 18
2018

1
death

Question-2: When an engineering judgment makes a lethal mistake, how do we assess blame?

Washington Post, March 21, 2018 page A7



PEDRO PORTAL/MIAMI HERALD/ASSOCIATED PRESS

The pedestrian bridge at Florida International University under construction. The 950-ton bridge collapsed Thursday, resulting in six deaths.

Thursday
Mar. 15
2018

6
deaths

Question-3: When a maintenance judgment makes a lethal mistake, how do we assess blame?

Southwest Airlines Flight Makes Emergency Landing At Philadelphia International Airport

Tuesday
Apr. 17
2018

1
death

1. Why is UQ important in Engineering ?

Ans. Because an engineer’s decision in design, manufacture, operation, and maintenance needs *estimates of stress with credible uncertainty bounds* for safe operation and failure prevention.

No. of slides	*Subtotal*
5	*5*

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1a. Why is UQ important in Engineering Research ?

Atomic Force Microscope

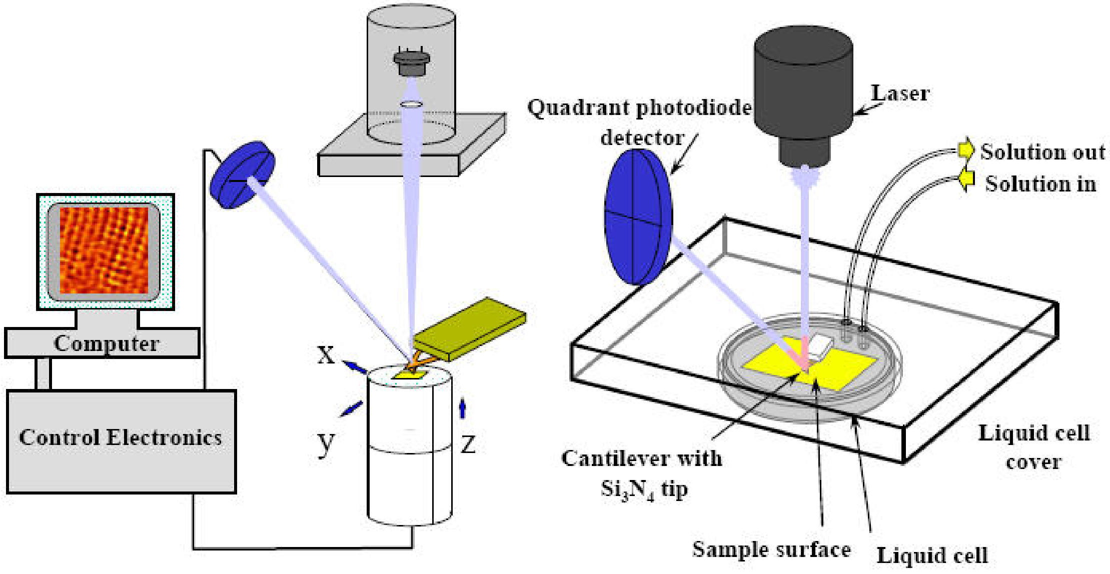
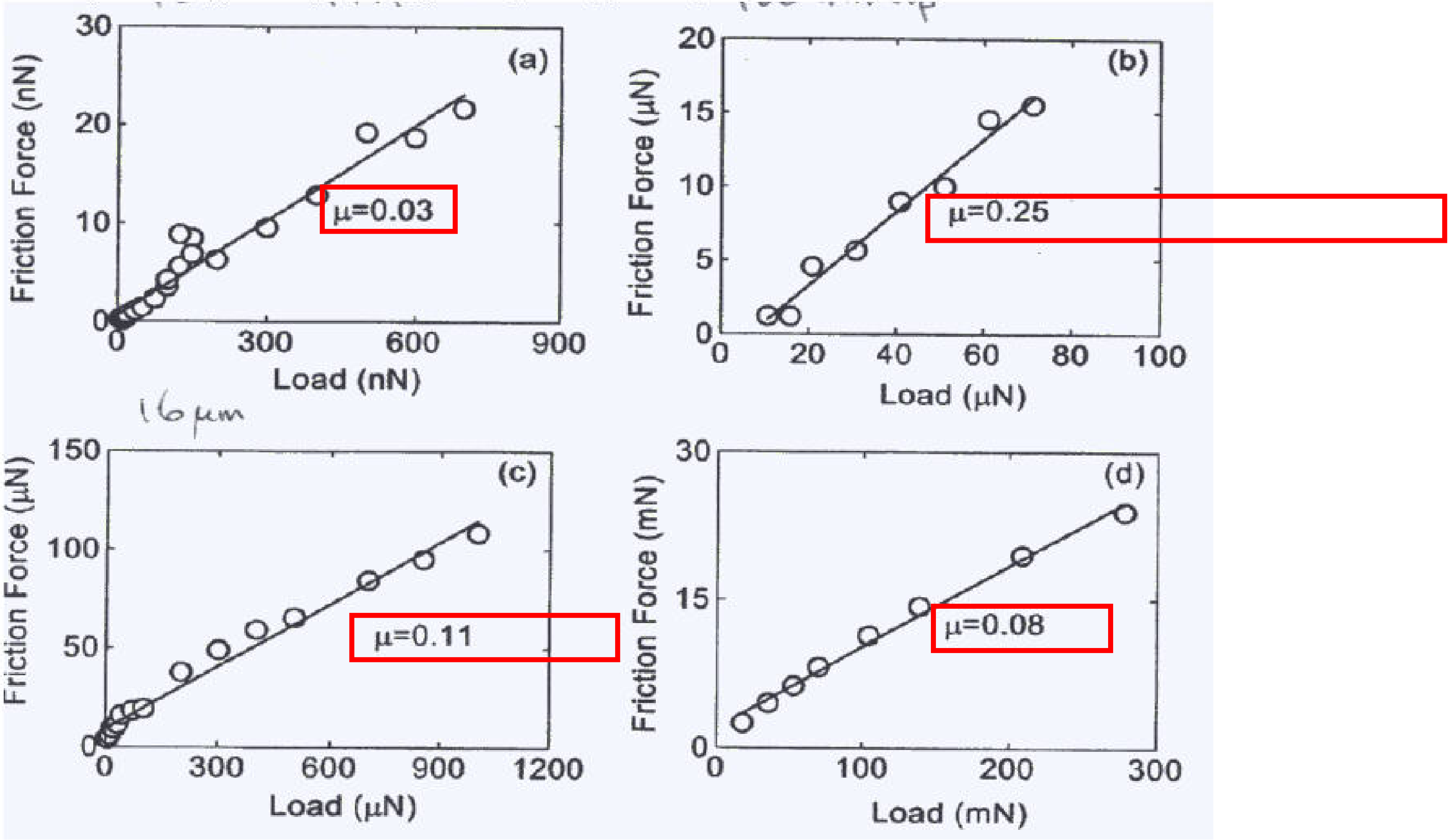
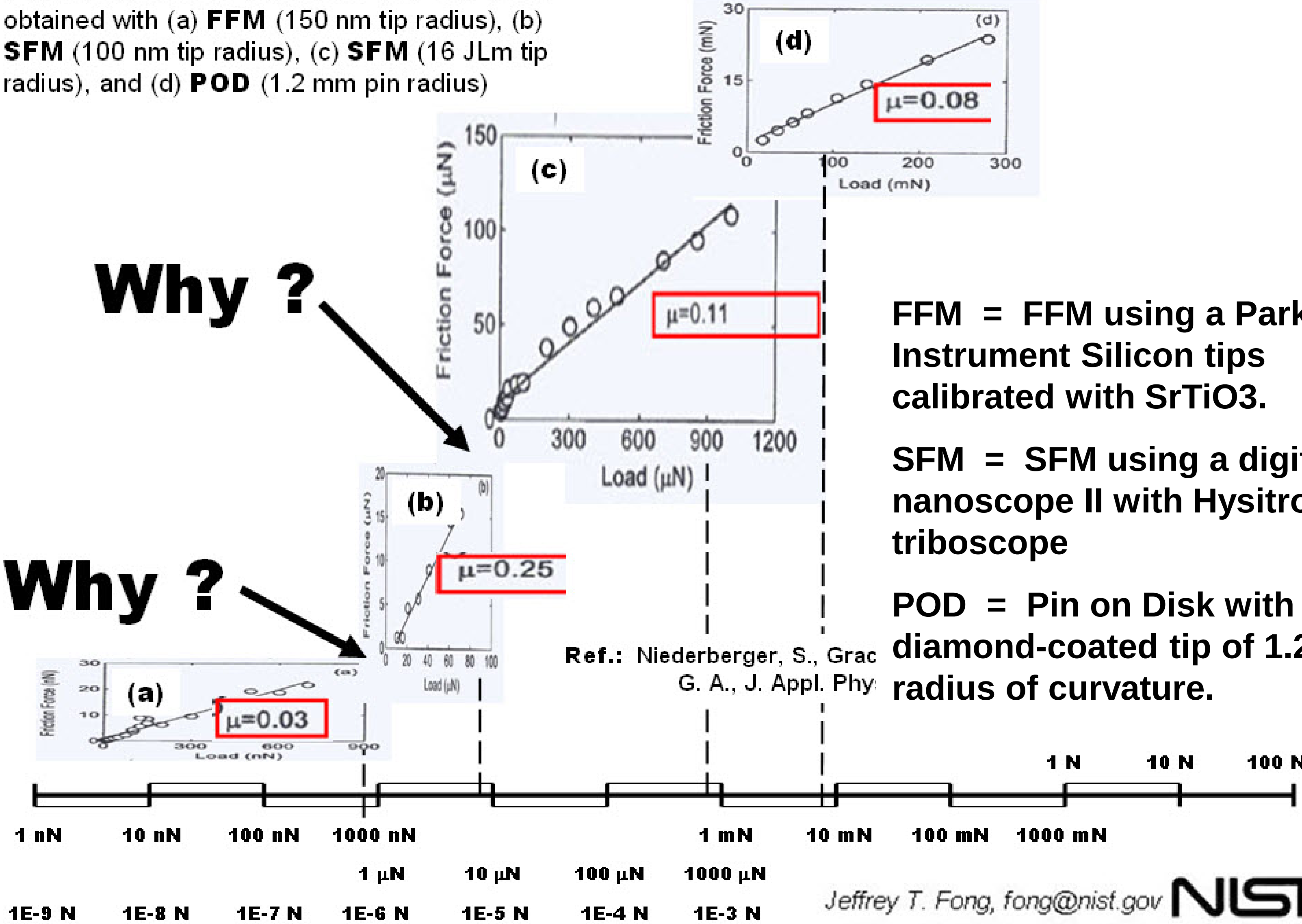


FIG. 6. Friction force vs load for Si(100) obtained with(a) FFM (150 nm tip radius), (b) SFM (100 nm tip radius), (c) SFM (16 JLm tip radius), and (d) POD (1.2 mm pin radius).



Ref.: **Niederberger, S., Gracias, D. H., Komvopoulos, K., Somorjai, G. A., *<u>J. Appl. Phys.,</u>* Vol. 87, No. 6, pp. 3143-3150 (2000).**

Friction force vs. load for Si(100) obtained with (a) **FFM** (150 nm tip radius), (b) **SFM** (100 nm tip radius), (c) **SFM** (16 JLm tip radius), and (d) **POD** (1.2 mm pin radius)



Why ?

Why ?

Ref.: Niederberger, S., Grac
G. A., J. Appl. Phy

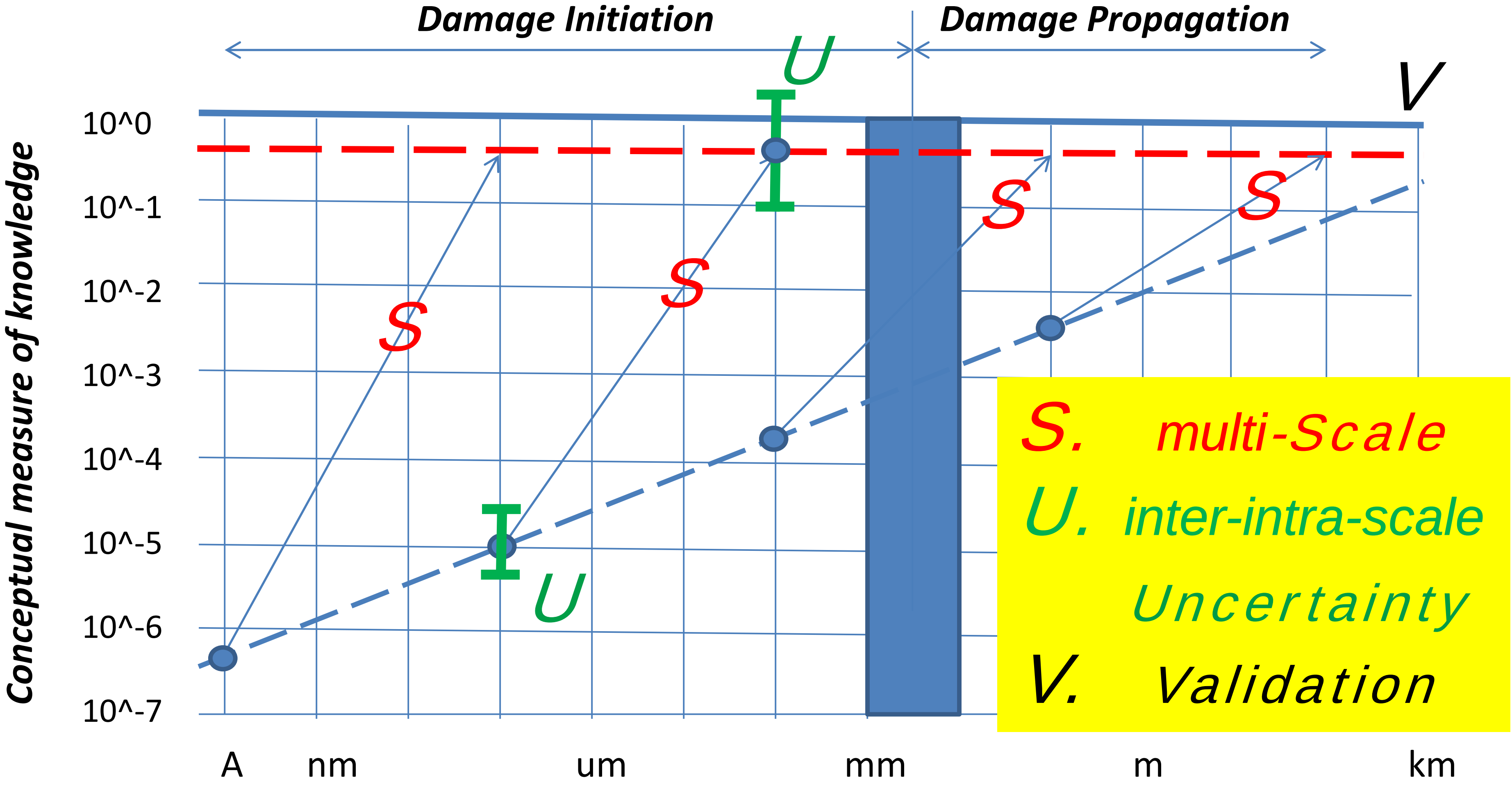
FFM = FFM using a Park Instrument Silicon tips calibrated with SrTiO3.

SFM = SFM using a digital nanoscope II with Hysitron triboscope

POD = Pin on Disk with a diamond-coated tip of 1.2 mm radius of curvature.

Jeffrey T. Fong, fong@nist.gov

1a. Why is UQ important in Engineering Research ?



1a. Why is UQ important in Engineering Research ?

Ans. Large scale simulations is often used to conduct fundamental research at the nano- and micro-scales. As the scale changes, forces and factors that are dominant at one scale may change at a different scale. Design of experiments tool allows one to rank the importance of factors at one scale, estimate uncertainty, re-formulate for a higher scale by discarding less dominant factors and add new factors to guide the design of a new experiment at a higher scale.

No. of slides	*Subtotal*
10	*10*

3. Six *Easy-to-use* Tools of Eng. UQ

3.1 Goodness-of-Fit (*GoF*) Test for *64* distributions.

3.2 Predictive Limits & Lower Tolerance Limit (*LTL*).

3.3 Linear Least SQuares (*LLSQ* - Regression).

3.4 Inter-laboratory comparison (*R&R*) and Variance Analysis (*ANOVA*).

Relatively new to Engineers

3.5 Design of Experiments (*DEX*).

3.6 Non-Linear Least SQuares with Logistic Function (*NLLSQ* - Lgs).

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What does one mean by Easy-to-use ?

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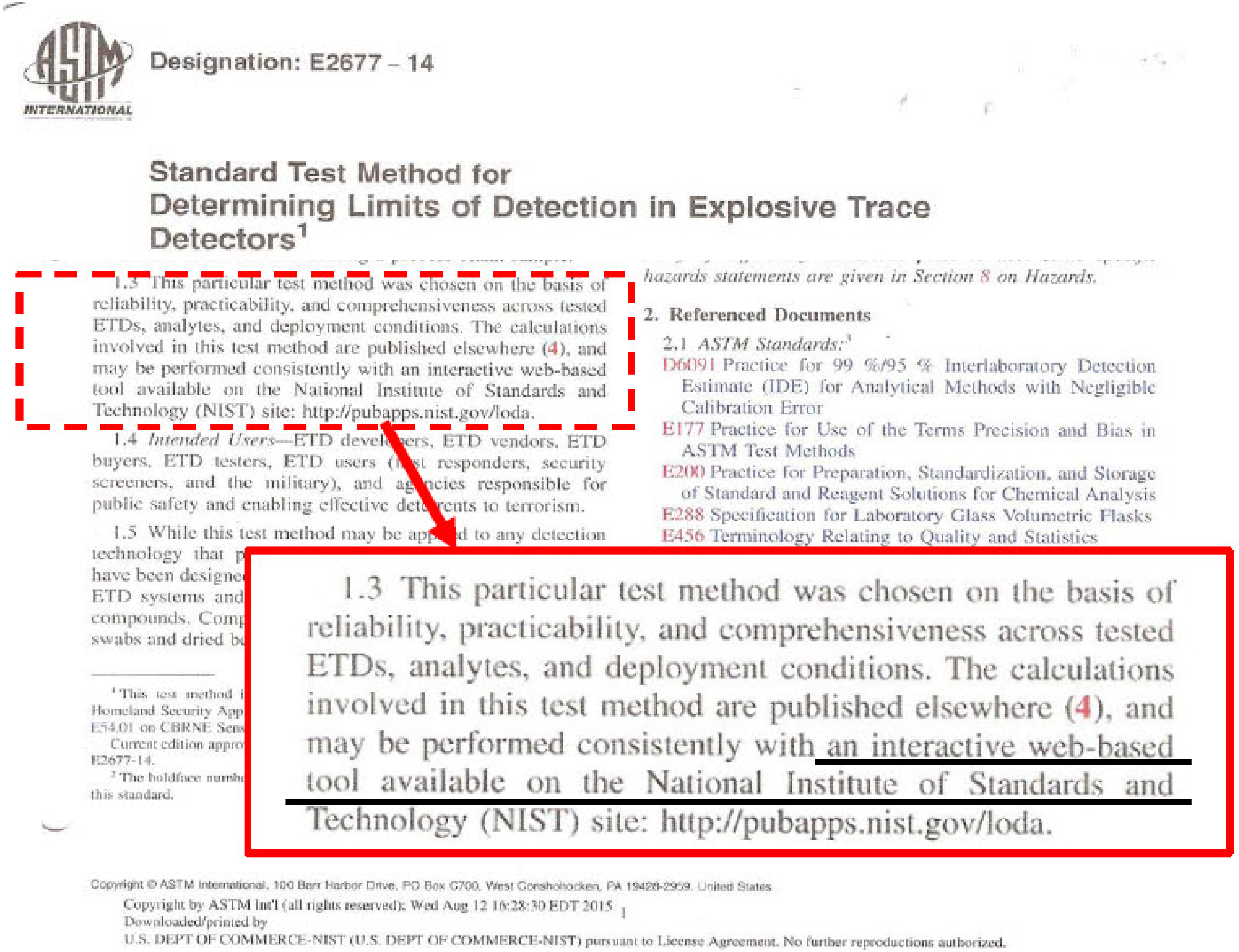
An Invited Presentation at UTA Research Institute, May 4, 2018, 12 noon (Prof. Ken reifsnider, host)



Designation: E2677 – 14

Standard Test Method for Determining Limits of Detection in Explosive Trace Detectors[1]

1.3 This particular test method was chosen on the basis of reliability, practicability, and comprehensiveness across tested ETDs, analytes, and deployment conditions. The calculations involved in this test method are published elsewhere (4), and may be performed consistently with an interactive web-based tool available on the National Institute of Standards and Technology (NIST) site: http://pubapps.nist.gov/loda.

1.4 *Intended Users*—ETD developers, ETD vendors, ETD buyers, ETD testers, ETD users (first responders, security screeners, and the military), and agencies responsible for public safety and enabling effective deterrents to terrorism.

1.5 While this test method may be applied to any detection technology that p
have been designe
ETD systems and
compounds. Com
swabs and dried b

hazards statements are given in Section 8 on Hazards.

2. Referenced Documents

2.1 *ASTM Standards:*[3]

D6091 Practice for 99 %/95 % Interlaboratory Detection Estimate (IDE) for Analytical Methods with Negligible Calibration Error

E177 Practice for Use of the Terms Precision and Bias in ASTM Test Methods

E200 Practice for Preparation, Standardization, and Storage of Standard and Reagent Solutions for Chemical Analysis

E288 Specification for Laboratory Glass Volumetric Flasks

E456 Terminology Relating to Quality and Statistics

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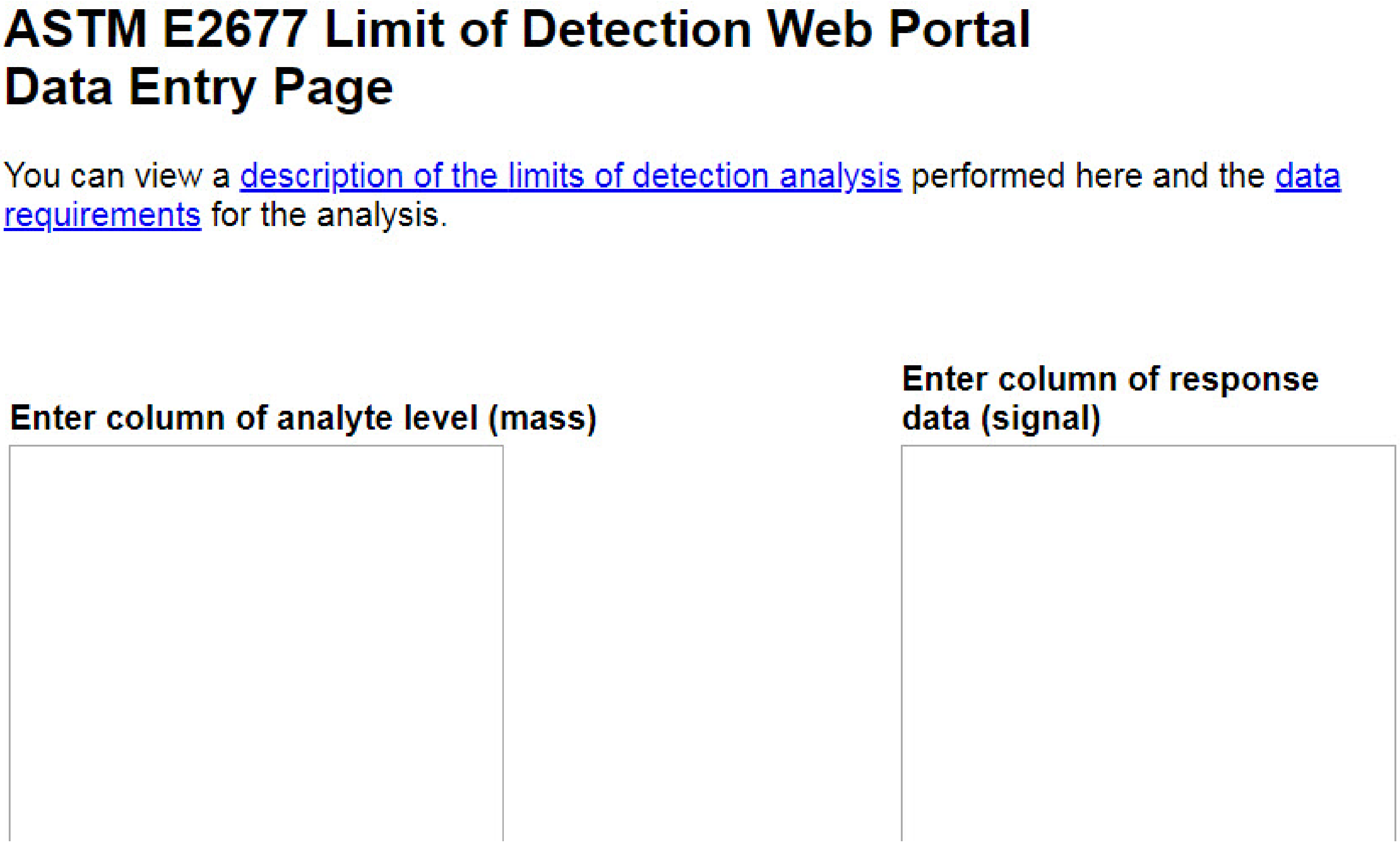
[1] This test method i
Homeland Security App
E54.01 on CBRNE Sens
Current edition appro
E2677-14.
[2] The boldface numb
this standard.

https://www-s.nist.gov/loda/index.html



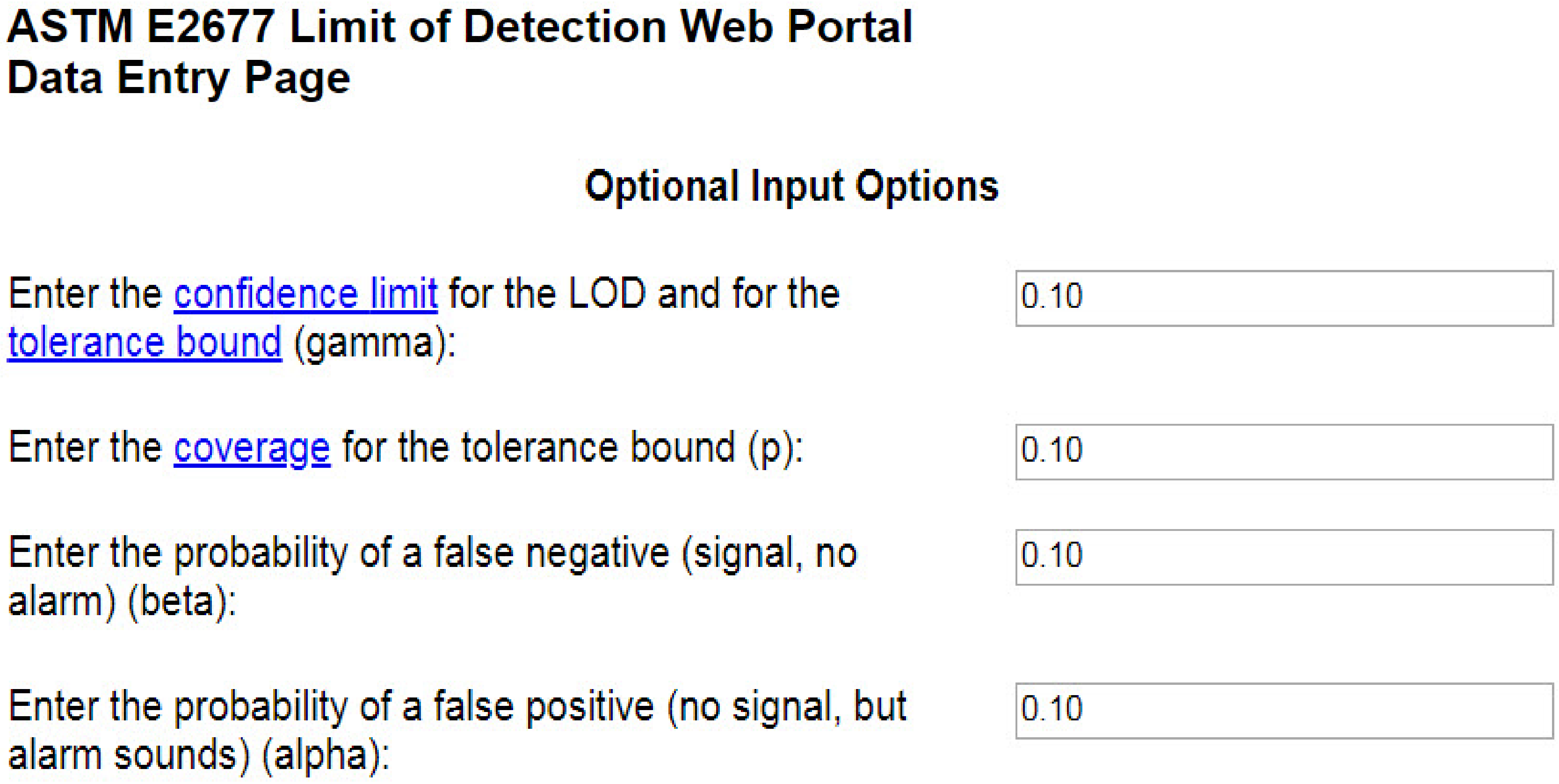
ASTM E2677 Limit of Detection Web Portal
Data Entry Page

You can view a description of the limits of detection analysis performed here and the data requirements for the analysis.

Enter column of analyte level (mass)

Enter column of response data (signal)

https://www-s.nist.gov/loda/index.html

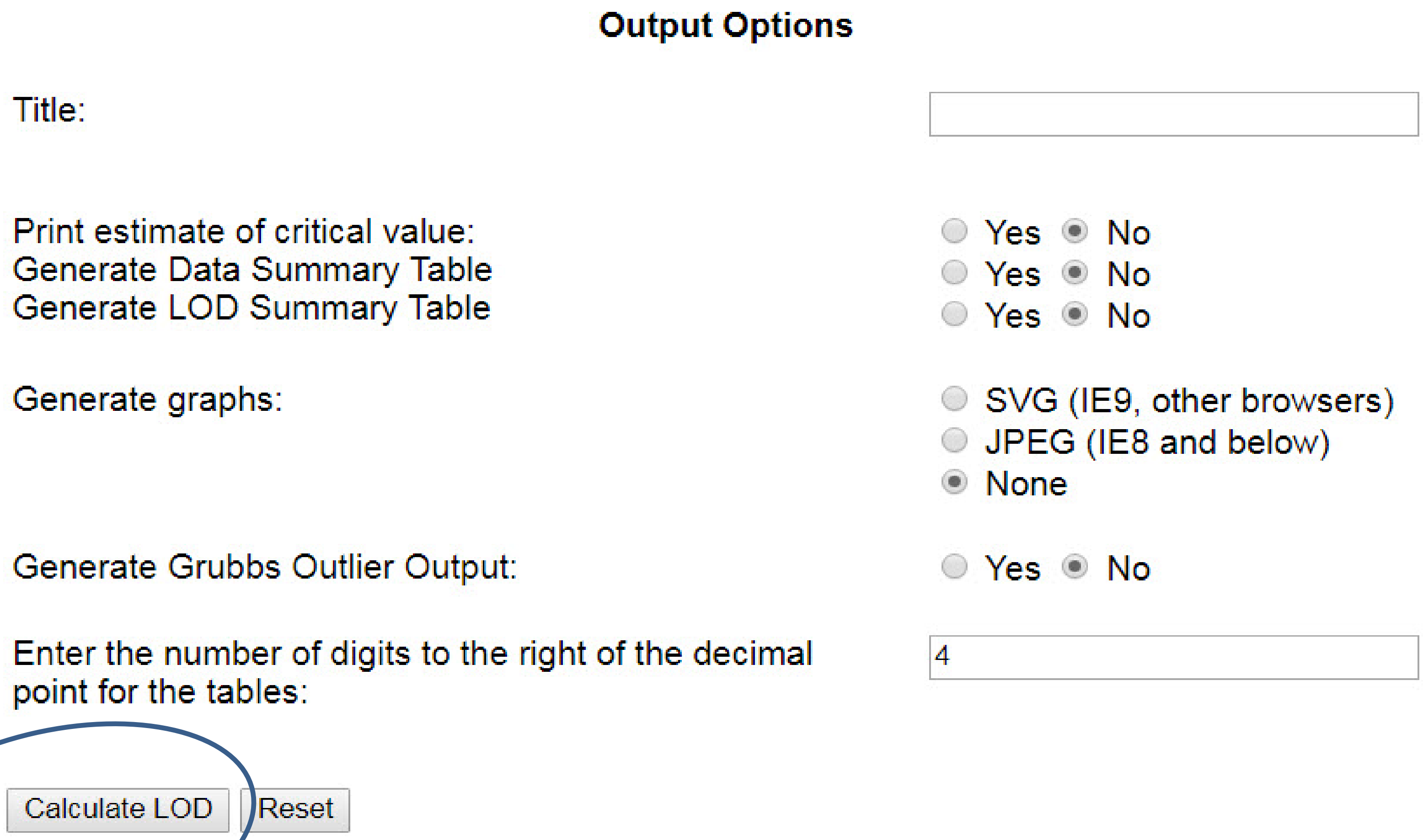


ASTM E2677 Limit of Detection Web Portal
Data Entry Page

Optional Input Options

Enter the confidence limit for the LOD and for the tolerance bound (gamma):	0.10
Enter the coverage for the tolerance bound (p):	0.10
Enter the probability of a false negative (signal, no alarm) (beta):	0.10
Enter the probability of a false positive (no signal, but alarm sounds) (alpha):	0.10

https://www-s.nist.gov/loda/index.html



Output Options

Title:

Print estimate of critical value: ○ Yes ● No
Generate Data Summary Table ○ Yes ● No
Generate LOD Summary Table ○ Yes ● No

Generate graphs: ○ SVG (IE9, other browsers) ○ JPEG (IE8 and below) ● None

Generate Grubbs Outlier Output: ○ Yes ● No

Enter the number of digits to the right of the decimal point for the tables: 4

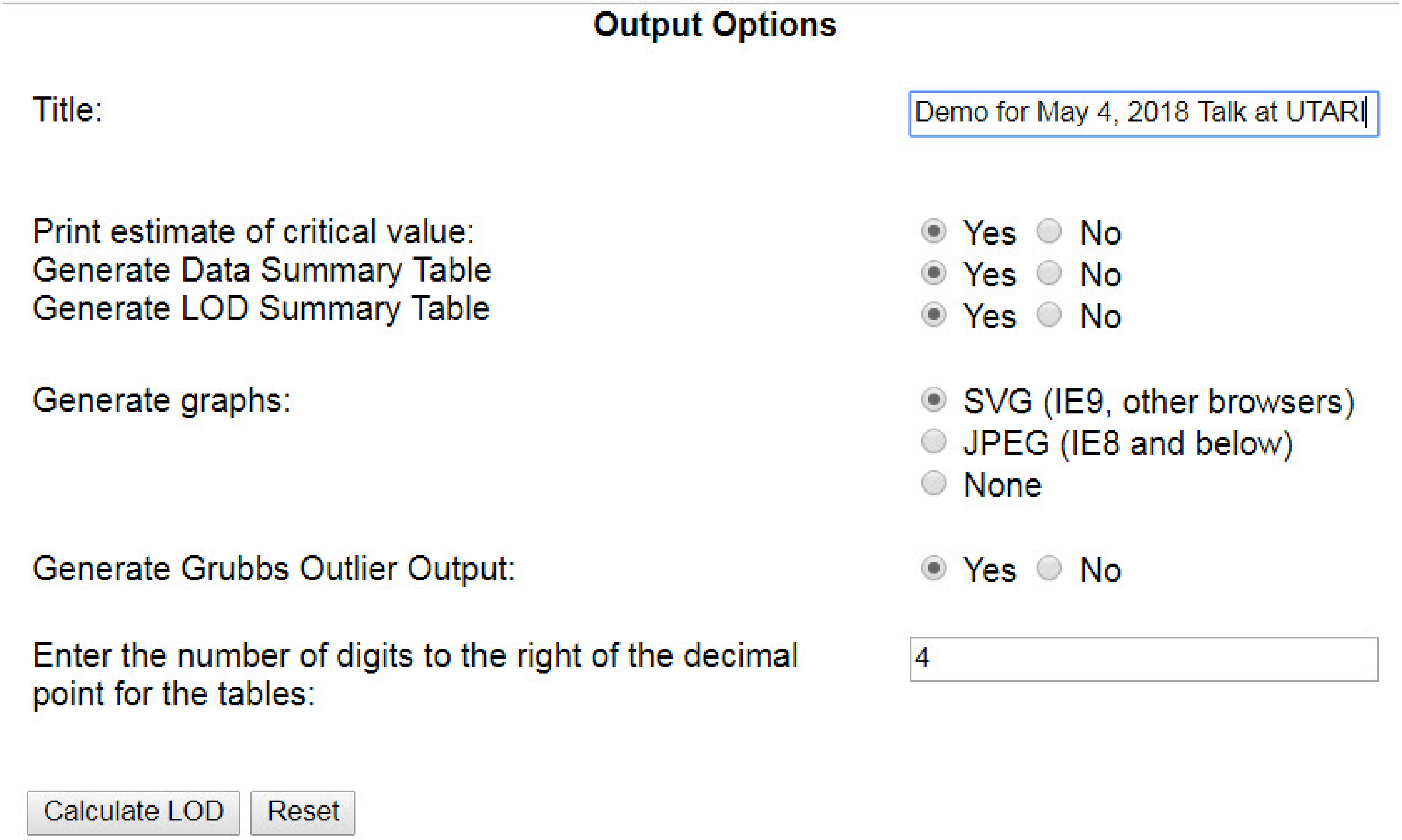
https://www-s.nist.gov/loda/index.html

Input Data File-1

```
0   0   0   0   0     0   0   0   0   0
0   0   0   0   0     0   0   0   0   0
0   0   0   0   0     0   0   0   0   0
0   0   0   0   0     0   0   0   0   0
0.1   0.1   0.1   0.1   0.1
0.1   0.1   0.1   0.1   0.1
0.1   0.1   0.1   0.1   0.1
0.3   0.3   0.3   0.3   0.3
0.3   0.3   0.3   0.3   0.3
0.3   0.3   0.3   0.3   0.3
1   1   1   1   1     1   1   1   1   1
1   1   1   1   1     3   3   3   3   3
3   3   3   3   3     3   3   3   3   3
```

Input Data File-2

```
  0     0    0    0    0      0     0     0 212 251
239     0    0    0    0      0   188     0 180   0
170 219 213    0    0      0     0 250 191 200
223 214 193    0    0      0     0 202 173   0
  0     0    0 294 174    242     0     0 272 203
189 211 212 239 282    198     0 191 223 218
236 177 274 244 342    222   237 261 279 284
255 248 338 426 279    280   264 313 351 321
400 283 349 357 344    749   614 739 861 711
751 654 695 689 611    726  1651 756 693 781
```



Output Options

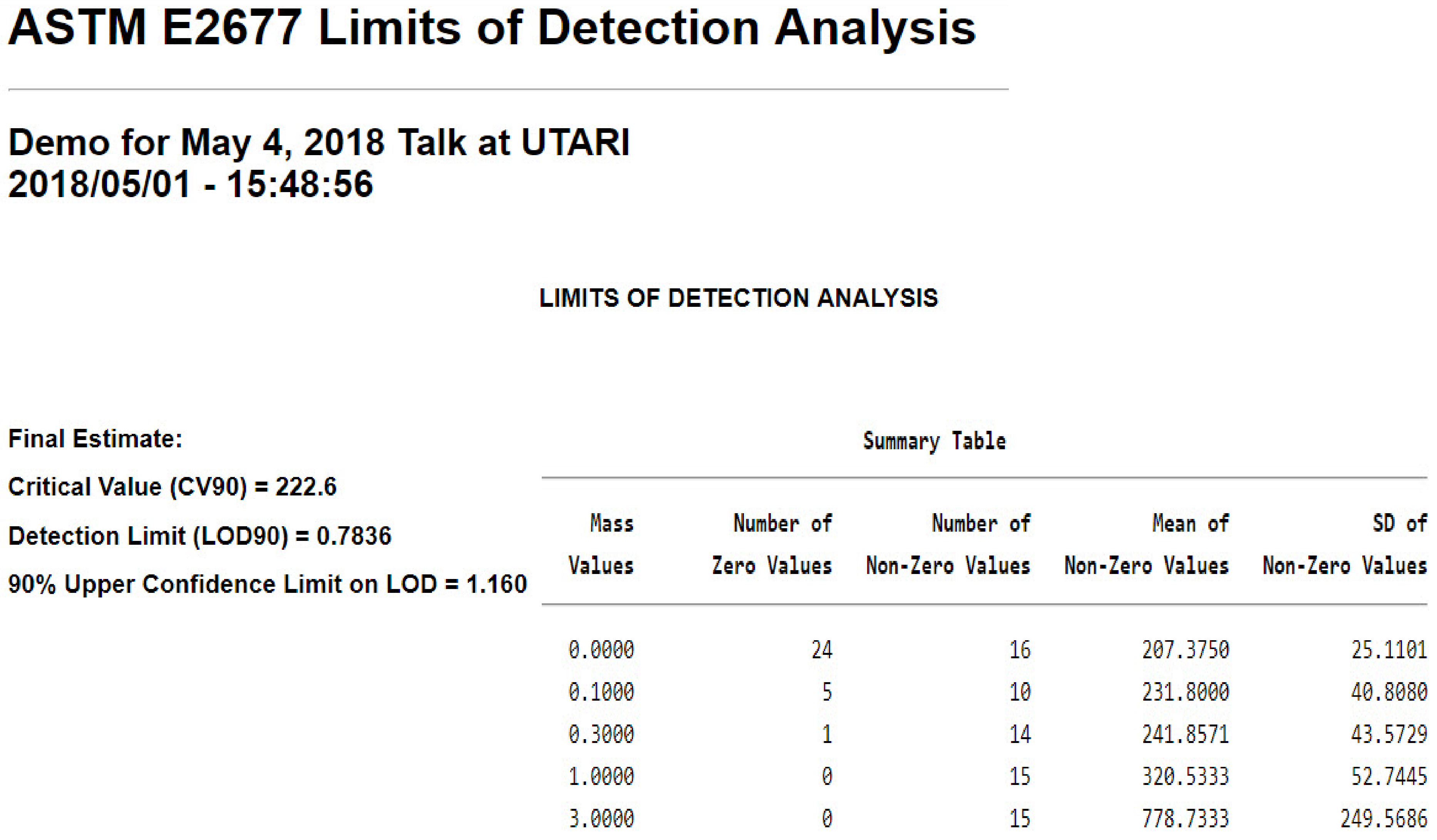
Title: Demo for May 4, 2018 Talk at UTARI

Print estimate of critical value: ● Yes ○ No
Generate Data Summary Table ● Yes ○ No
Generate LOD Summary Table ● Yes ○ No

Generate graphs:
● SVG (IE9, other browsers)
○ JPEG (IE8 and below)
○ None

Generate Grubbs Outlier Output: ● Yes ○ No

Enter the number of digits to the right of the decimal point for the tables: 4



ASTM E2677 Limits of Detection Analysis

Demo for May 4, 2018 Talk at UTARI
2018/05/01 - 15:48:56

LIMITS OF DETECTION ANALYSIS

Final Estimate:

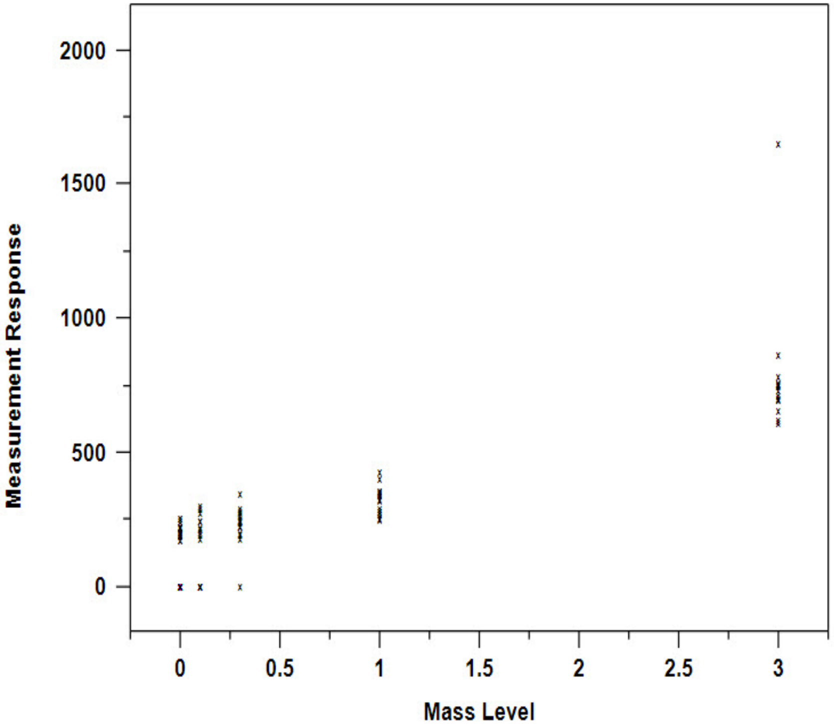
Critical Value (CV90) = 222.6

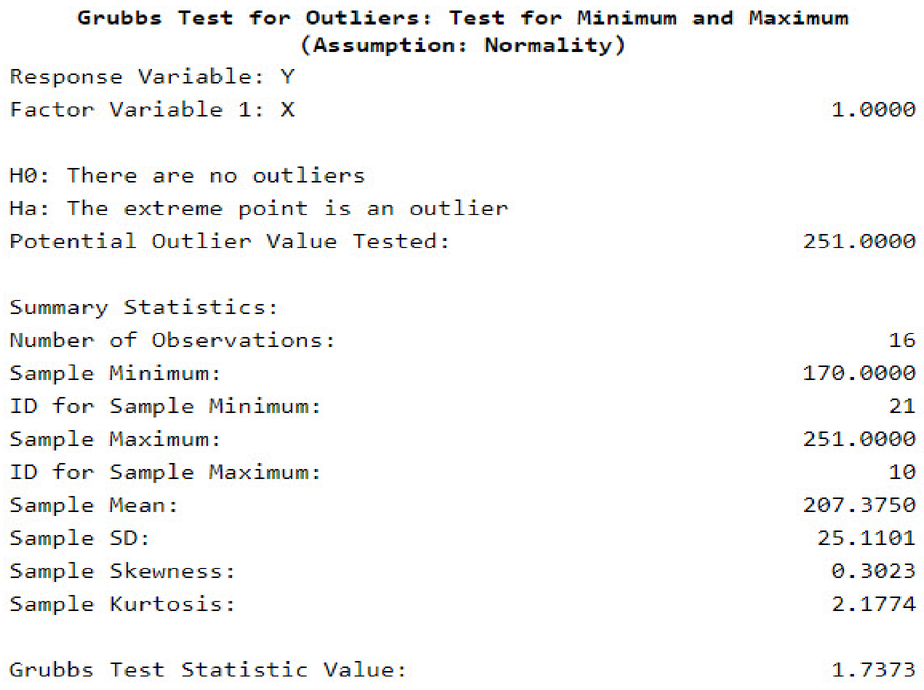
Detection Limit (LOD90) = 0.7836

90% Upper Confidence Limit on LOD = 1.160

Summary Table

Mass Values	Number of Zero Values	Number of Non-Zero Values	Mean of Non-Zero Values	SD of Non-Zero Values
0.0000	24	16	207.3750	25.1101
0.1000	5	10	231.8000	40.8080
0.3000	1	14	241.8571	43.5729
1.0000	0	15	320.5333	52.7445
3.0000	0	15	778.7333	249.5686



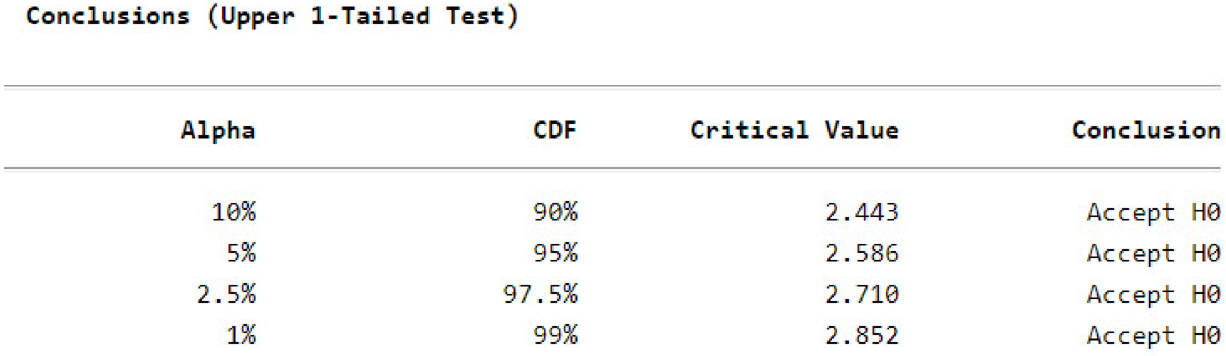


```
           Grubbs Test for Outliers: Test for Minimum and Maximum
                         (Assumption: Normality)
Response Variable: Y
Factor Variable 1: X                                       1.0000

H0: There are no outliers
Ha: The extreme point is an outlier
Potential Outlier Value Tested:                          251.0000

Summary Statistics:
Number of Observations:                                        16
Sample Minimum:                                          170.0000
ID for Sample Minimum:                                         21
Sample Maximum:                                          251.0000
ID for Sample Maximum:                                         10
Sample Mean:                                             207.3750
Sample SD:                                                25.1101
Sample Skewness:                                           0.3023
Sample Kurtosis:                                           2.1774

Grubbs Test Statistic Value:                               1.7373
```



```
Conclusions (Upper 1-Tailed Test)

---------------------------------------------------------------------
           Alpha               CDF      Critical Value     Conclusion
---------------------------------------------------------------------
             10%               90%               2.443      Accept H0
              5%               95%               2.586      Accept H0
            2.5%             97.5%               2.710      Accept H0
              1%               99%               2.852      Accept H0
```

No. of slides	*Subtotal*
10	*20*

3. Six *Easy-to-use* Tools of Eng. UQ

3.1 Goodness-of-Fit (*GoF*) Test for *64* distributions.

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Relatively new to Engineers

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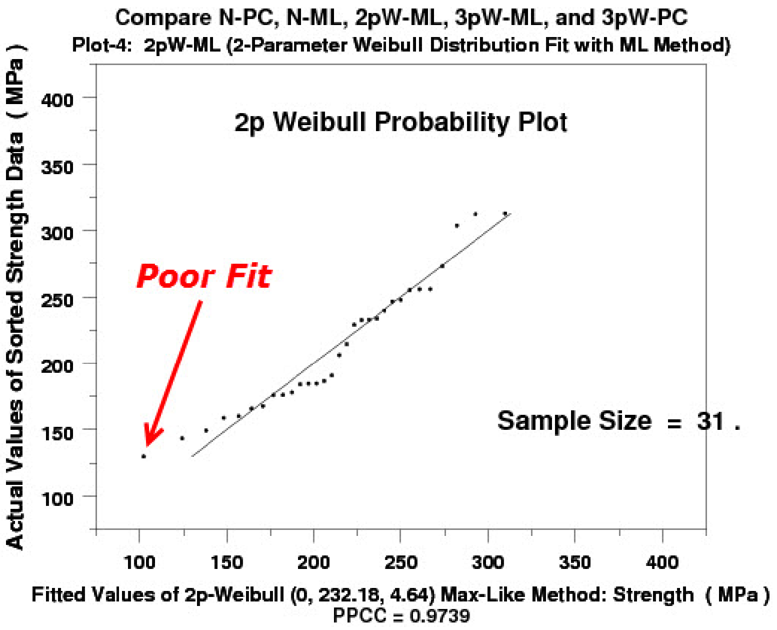
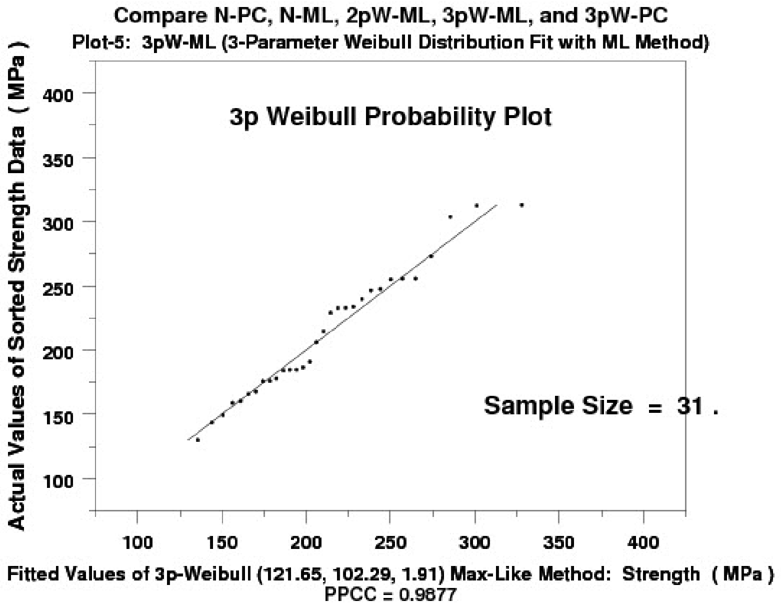
3. Six Tools of Eng. UQ

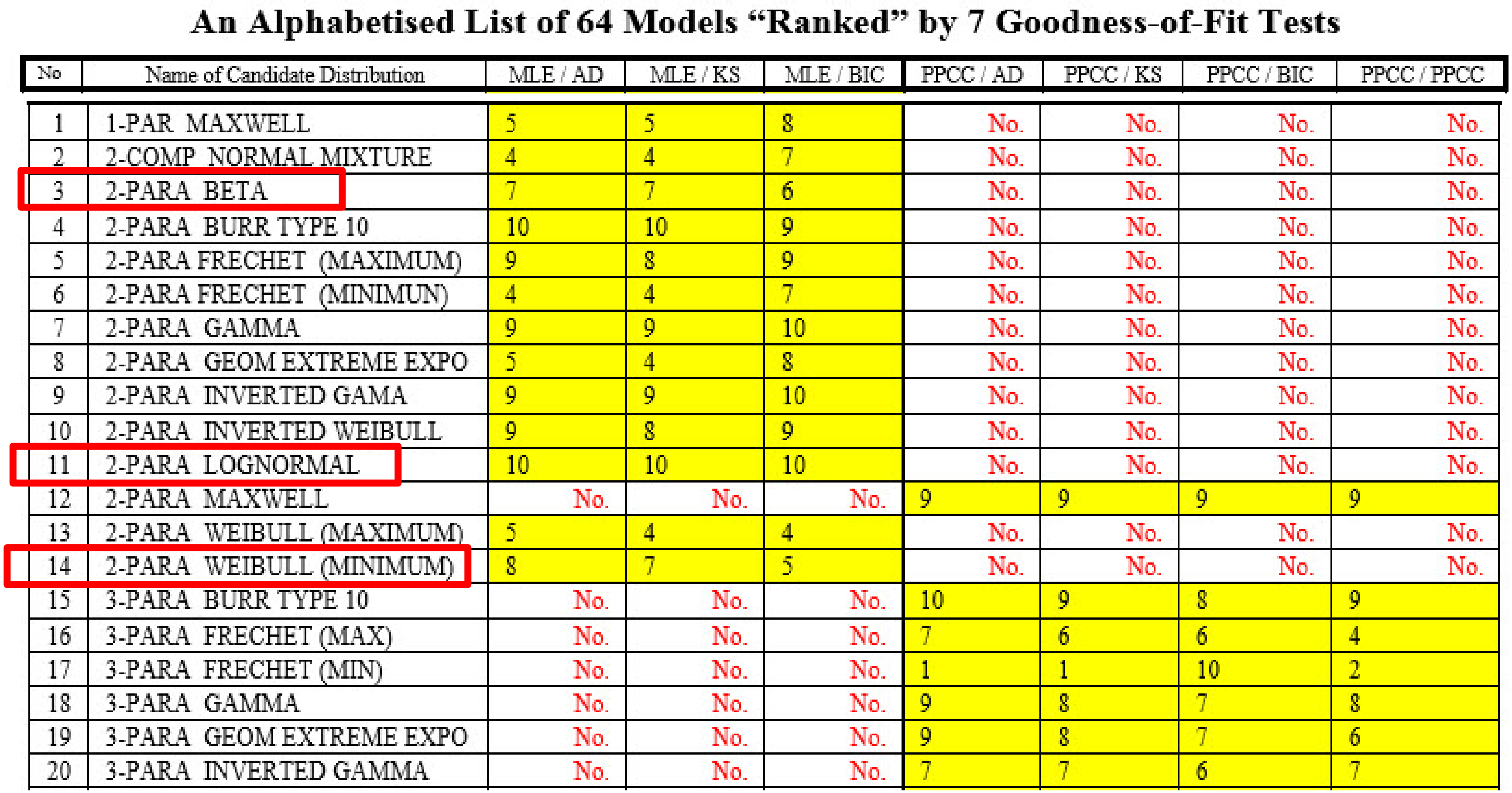
Tool-1

Goodness-of-Fit (*GoF*) Test for *64* distributions.





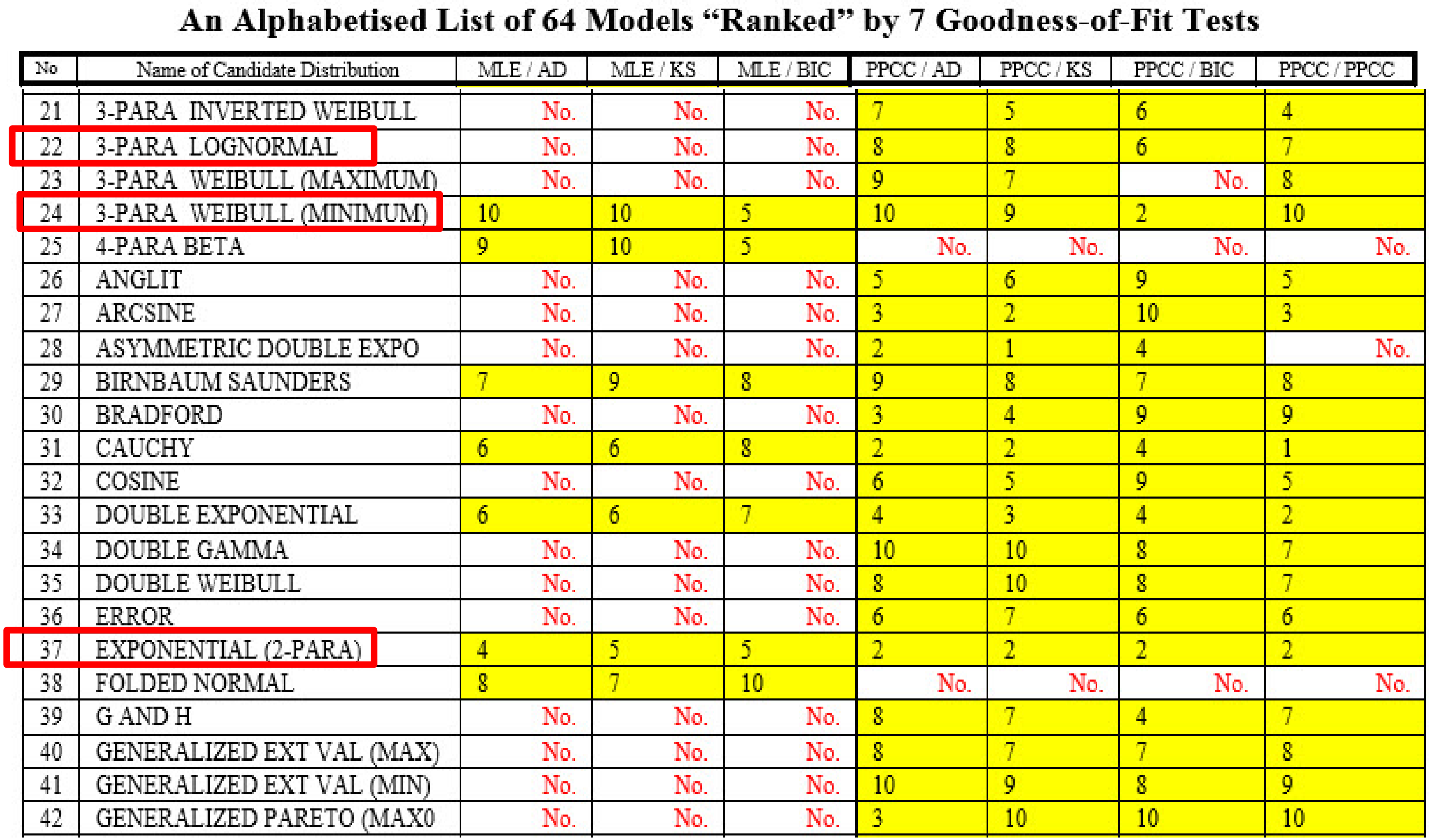
Tool-1 Goodness-of-Fit (*GoF*) Test for *64* distributions.



An Alphabetised List of 64 Models “Ranked” by 7 Goodness-of-Fit Tests

No	Name of Candidate Distribution	MLE / AD	MLE / KS	MLE / BIC	PPCC / AD	PPCC / KS	PPCC / BIC	PPCC / PPCC
1	1-PAR MAXWELL	5	5	8	No.	No.	No.	No.
2	2-COMP NORMAL MIXTURE	4	4	7	No.	No.	No.	No.
3	2-PARA BETA	7	7	6	No.	No.	No.	No.
4	2-PARA BURR TYPE 10	10	10	9	No.	No.	No.	No.
5	2-PARA FRECHET (MAXIMUM)	9	8	9	No.	No.	No.	No.
6	2-PARA FRECHET (MINIMUN)	4	4	7	No.	No.	No.	No.
7	2-PARA GAMMA	9	9	10	No.	No.	No.	No.
8	2-PARA GEOM EXTREME EXPO	5	4	8	No.	No.	No.	No.
9	2-PARA INVERTED GAMA	9	9	10	No.	No.	No.	No.
10	2-PARA INVERTED WEIBULL	9	8	9	No.	No.	No.	No.
11	2-PARA LOGNORMAL	10	10	10	No.	No.	No.	No.
12	2-PARA MAXWELL	No.	No.	No.	9	9	9	9
13	2-PARA WEIBULL (MAXIMUM)	5	4	4	No.	No.	No.	No.
14	2-PARA WEIBULL (MINIMUM)	8	7	5	No.	No.	No.	No.
15	3-PARA BURR TYPE 10	No.	No.	No.	10	9	8	9
16	3-PARA FRECHET (MAX)	No.	No.	No.	7	6	6	4
17	3-PARA FRECHET (MIN)	No.	No.	No.	1	1	10	2
18	3-PARA GAMMA	No.	No.	No.	9	8	7	8
19	3-PARA GEOM EXTREME EXPO	No.	No.	No.	9	8	7	6
20	3-PARA INVERTED GAMMA	No.	No.	No.	7	7	6	7

3.1 Goodness-of-Fit (*GoF*) Test for *64* distributions.



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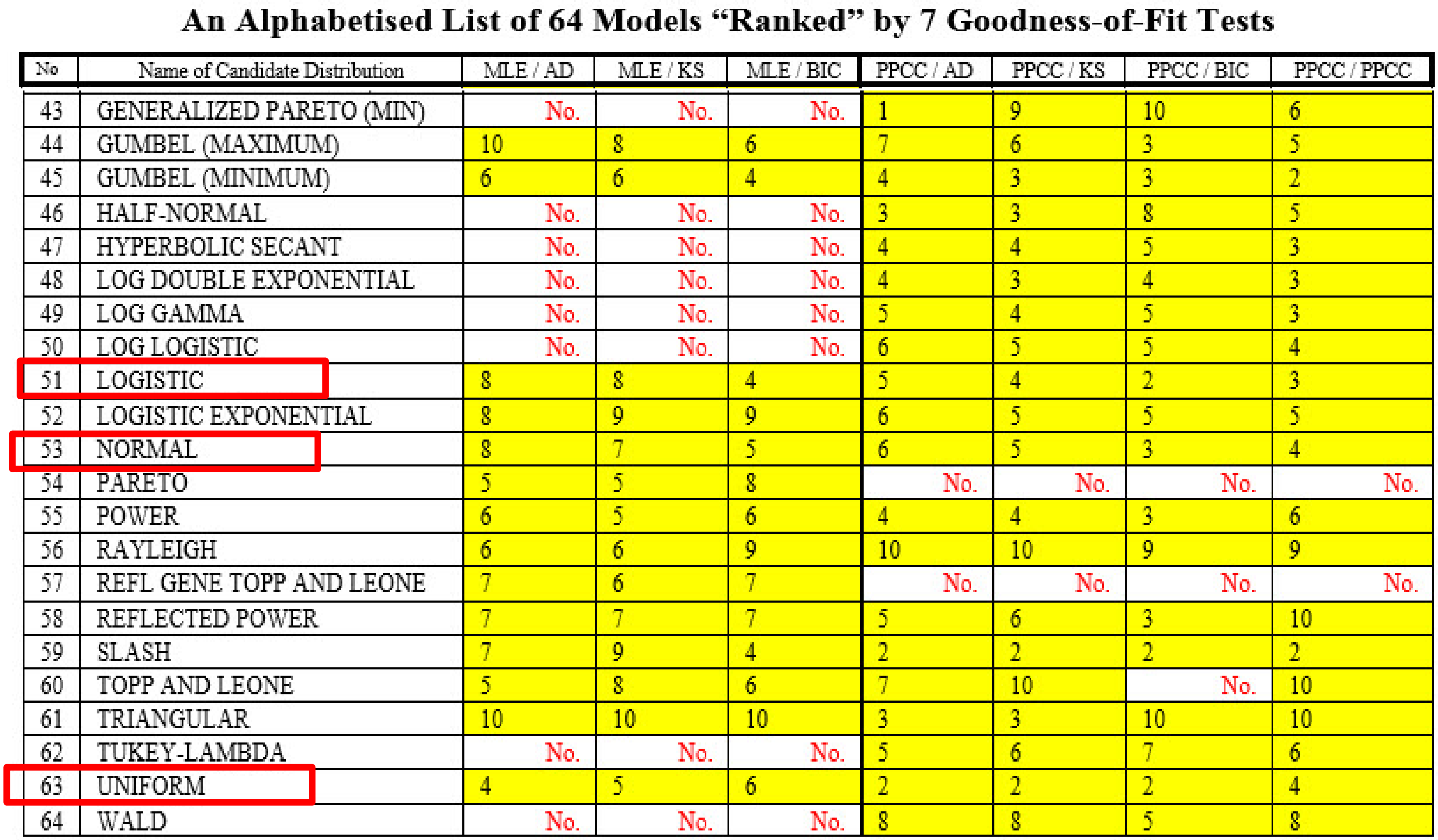
21	3-PARA INVERTED WEIBULL	No.	No.	No.	7	5	6	4
22	3-PARA LOGNORMAL	No.	No.	No.	8	8	6	7
23	3-PARA WEIBULL (MAXIMUM)	No.	No.	No.	9	7	No.	8
24	3-PARA WEIBULL (MINIMUM)	10	10	5	10	9	2	10
25	4-PARA BETA	9	10	5	No.	No.	No.	No.
26	ANGLIT	No.	No.	No.	5	6	9	5
27	ARCSINE	No.	No.	No.	3	2	10	3
28	ASYMMETRIC DOUBLE EXPO	No.	No.	No.	2	1	4	No.
29	BIRNBAUM SAUNDERS	7	9	8	9	8	7	8
30	BRADFORD	No.	No.	No.	3	4	9	9
31	CAUCHY	6	6	8	2	2	4	1
32	COSINE	No.	No.	No.	6	5	9	5
33	DOUBLE EXPONENTIAL	6	6	7	4	3	4	2
34	DOUBLE GAMMA	No.	No.	No.	10	10	8	7
35	DOUBLE WEIBULL	No.	No.	No.	8	10	8	7
36	ERROR	No.	No.	No.	6	7	6	6
37	EXPONENTIAL (2-PARA)	4	5	5	2	2	2	2
38	FOLDED NORMAL	8	7	10	No.	No.	No.	No.
39	G AND H	No.	No.	No.	8	7	4	7
40	GENERALIZED EXT VAL (MAX)	No.	No.	No.	8	7	7	8
41	GENERALIZED EXT VAL (MIN)	No.	No.	No.	10	9	8	9
42	GENERALIZED PARETO (MAX0	No.	No.	No.	3	10	10	10

3.1 Goodness-of-Fit (*GoF*) Test for *64* distributions.



An Alphabetised List of 64 Models "Ranked" by 7 Goodness-of-Fit Tests

No	Name of Candidate Distribution	MLE / AD	MLE / KS	MLE / BIC	PPCC / AD	PPCC / KS	PPCC / BIC	PPCC / PPCC
43	GENERALIZED PARETO (MIN)	No.	No.	No.	1	9	10	6
44	GUMBEL (MAXIMUM)	10	8	6	7	6	3	5
45	GUMBEL (MINIMUM)	6	6	4	4	3	3	2
46	HALF-NORMAL	No.	No.	No.	3	3	8	5
47	HYPERBOLIC SECANT	No.	No.	No.	4	4	5	3
48	LOG DOUBLE EXPONENTIAL	No.	No.	No.	4	3	4	3
49	LOG GAMMA	No.	No.	No.	5	4	5	3
50	LOG LOGISTIC	No.	No.	No.	6	5	5	4
51	LOGISTIC	8	8	4	5	4	2	3
52	LOGISTIC EXPONENTIAL	8	9	9	6	5	5	5
53	NORMAL	8	7	5	6	5	3	4
54	PARETO	5	5	8	No.	No.	No.	No.
55	POWER	6	5	6	4	4	3	6
56	RAYLEIGH	6	6	9	10	10	9	9
57	REFL GENE TOPP AND LEONE	7	6	7	No.	No.	No.	No.
58	REFLECTED POWER	7	7	7	5	6	3	10
59	SLASH	7	9	4	2	2	2	2
60	TOPP AND LEONE	5	8	6	7	10	No.	10
61	TRIANGULAR	10	10	10	3	3	10	10
62	TUKEY-LAMBDA	No.	No.	No.	5	6	7	6
63	UNIFORM	4	5	6	2	2	2	4
64	WALD	No.	No.	No.	8	8	5	8

3. Six Tools of Eng. UQ

Tool-2

Predictive Limits

&

Lower Tolerance Limit (*LTL*).

Tool-2 Predictive Limits & Lower Tolerance Limit (*LTL*).

3 intervals for Error Estimation

Coni < **Predi** < **Toli**.

coni = ($m - d1 * s$, $m + d1 * s$), with

$d1 = t(\alpha/2, n\text{-}1) / \mathrm{sqrt}(n)$.

Predi: $d2 = t(\alpha/2, n\text{-}1) * \mathrm{sqrt}(1 + 1/n)$.

Toli: $d3 = r * u$,

$r = r(p, n)$, and $u = u(1\text{-}\alpha, df)$

n = sample size.

1 - α = confidence.

t = t-distribution.

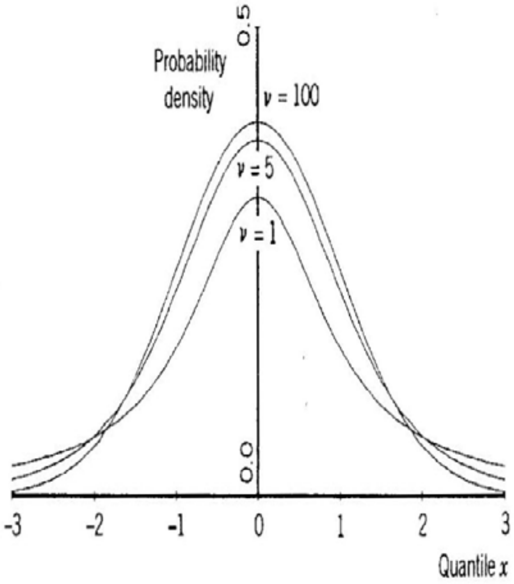
df = n – 1 .

p = coverage.

3.2 Predictive Limits & Lower Tolerance Limit (*LTL*).

Q.-3.2.1 What is a t-distribution ?

Let’s graph it **[5]:**



For ν = n – 1 = df, the p.d.f. (probability density function) of a t-distribution (t ; ν) is given by f (x; ν) =

$$\frac{\{\Gamma[(\nu+1)/2]\}}{(\pi\nu)^{1/2}\Gamma(\nu/2)[1+(x^2/\nu)]^{(\nu+1)/2}}$$

where its mean = 0 , ($\nu > 1$),

and its variance = $\nu / (\nu - 2)$, ($\nu > 2$).

3.2 Predictive Limits & Lower Tolerance Limit (*LTL*).

Q.-3.2.2 How does one calculate a confidence interval (Coni) ?

An example of a 10-data set of the ultimate tensile strength (MPa) of a material X is given below by an ordered set of 10 numbers:

73, 76, 80, 90, 100 , 100, 110, 120, 124, 127,

where n = 10, ***m = 100****, and s.d. = 20. From the t-table, t (0.025, 9) = 2.262. From the formula for d1 , d1 = 2.262 * 20 * sqrt (1/10) = 14.31. Therefore, the confidence interval at 95 % C.L. is,* ***(86, 114).***

Q.-3.2.3 How does one calculate a predictive interval (Predi) ?

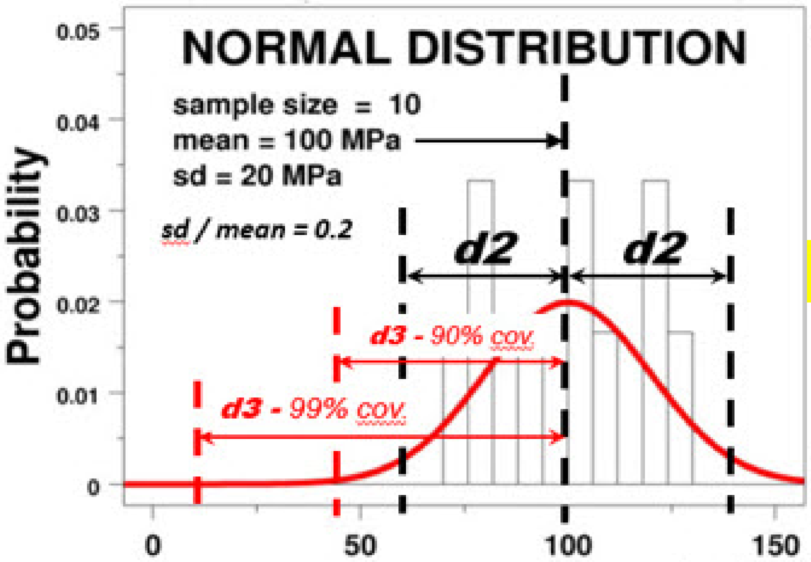
An example of a 10-data set of the ultimate tensile strength (MPa) of a material X is given below by an ordered set of 10 numbers:

73, 76, 80, 90, 100 , 100, 110, 120, 124, 127,

where n = 10, ***m = 100****, and s.d. = 20. From the t-table,*
t (0.025, 9) = 2.262. From the formula for d2 ,
*d2 = 2.262 * 20 * sqrt (1+1/10) = 47.45. Therefore,*
predictive interval at 95 % C.L. is, ***(53, [data set] 147).***

Note: Predictive interval is always wider than Confidence Interval.

3.2 Predictive Limits & Lower Tolerance Limit (*LTL*).



Confidence Interval = *(m - d₁, m + d₁)*

$$d_1 = t(0.025; n-1) * s.d. * \sqrt{\frac{1}{n}}$$

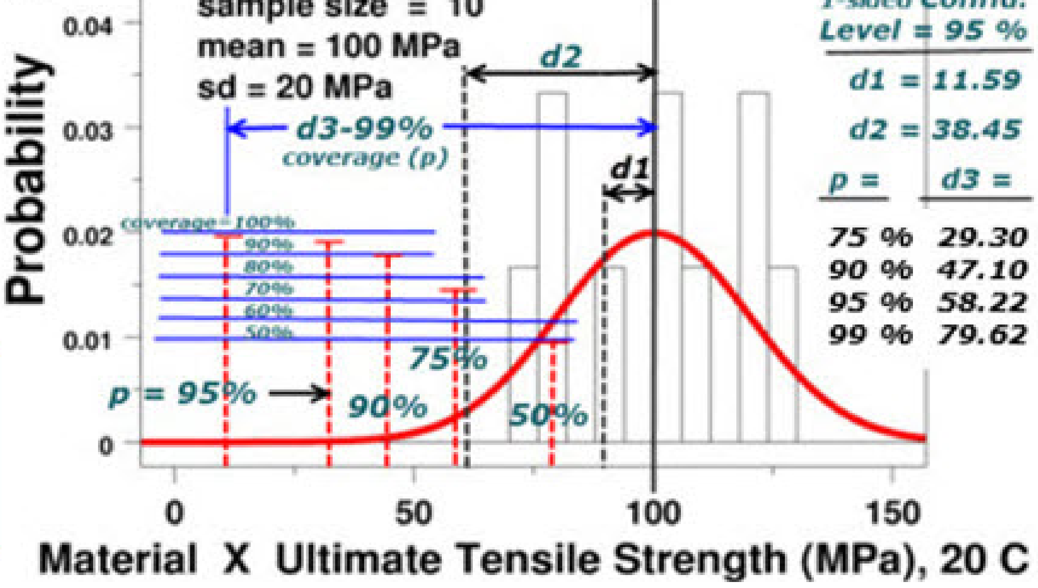
Predictive Lower Limit = mean – d2

$$d_2 = t(0.025; n-1) * s.d. * \sqrt{1 + \frac{1}{n}}$$

Tolerance Lower Limits
= mean – d3
(Population space)

$$d_3 = \gamma * s.d. * \mu$$

where γ depends on p , the coverage of the population, and m depends on n-1 and the conf. level.



1-sided Confid. Level = 95 %

d1 = 11.59

d2 = 38.45

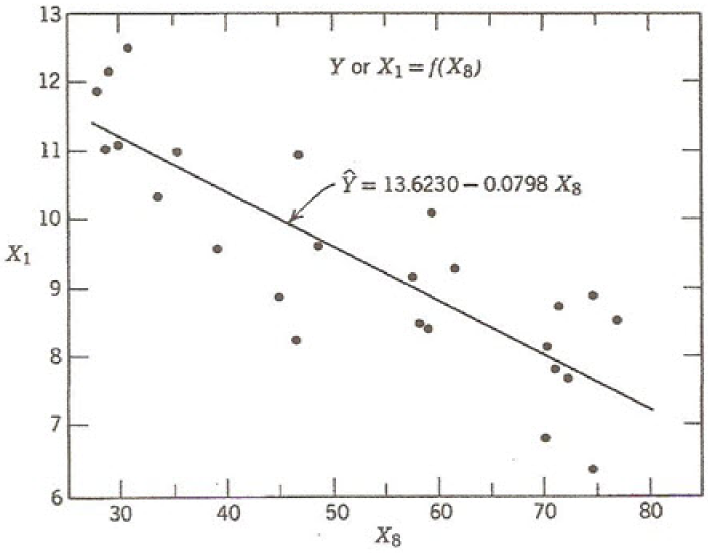
p =	*d3 =*
75 %	*29.30*
90 %	*47.10*
95 %	*58.22*
99 %	*79.62*

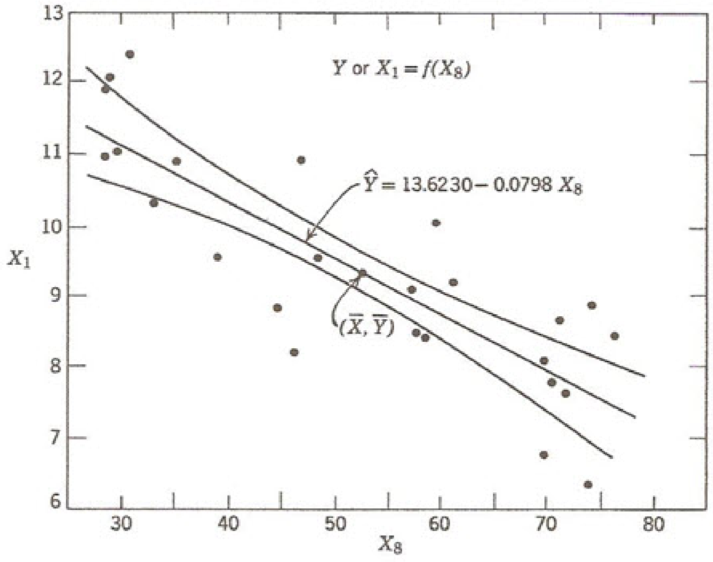
3. Six Tools of Eng. UQ

Tool-3

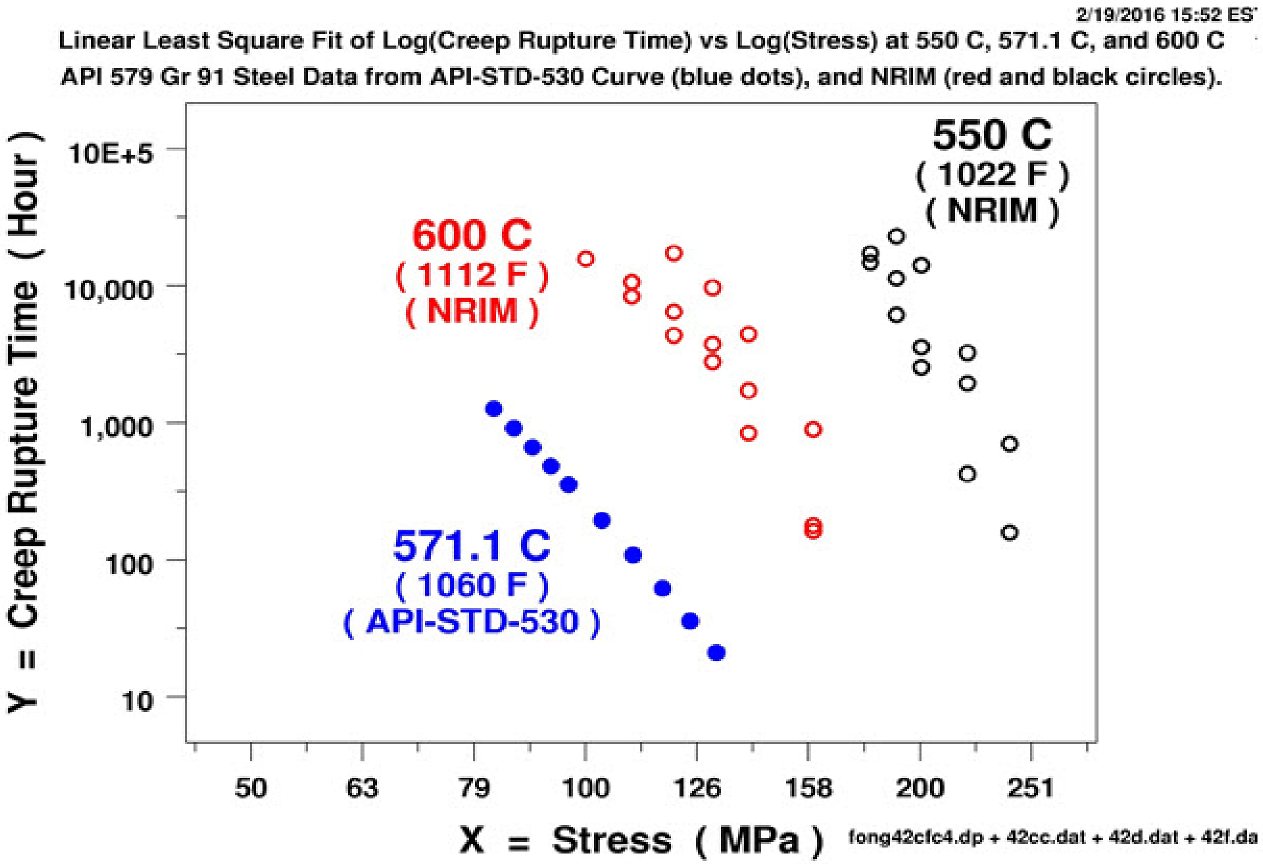
Linear Least Squares

(*LLSQ* - Regression).





Tool-3 Linear Least SQuares (*LLSQ* - Regression).



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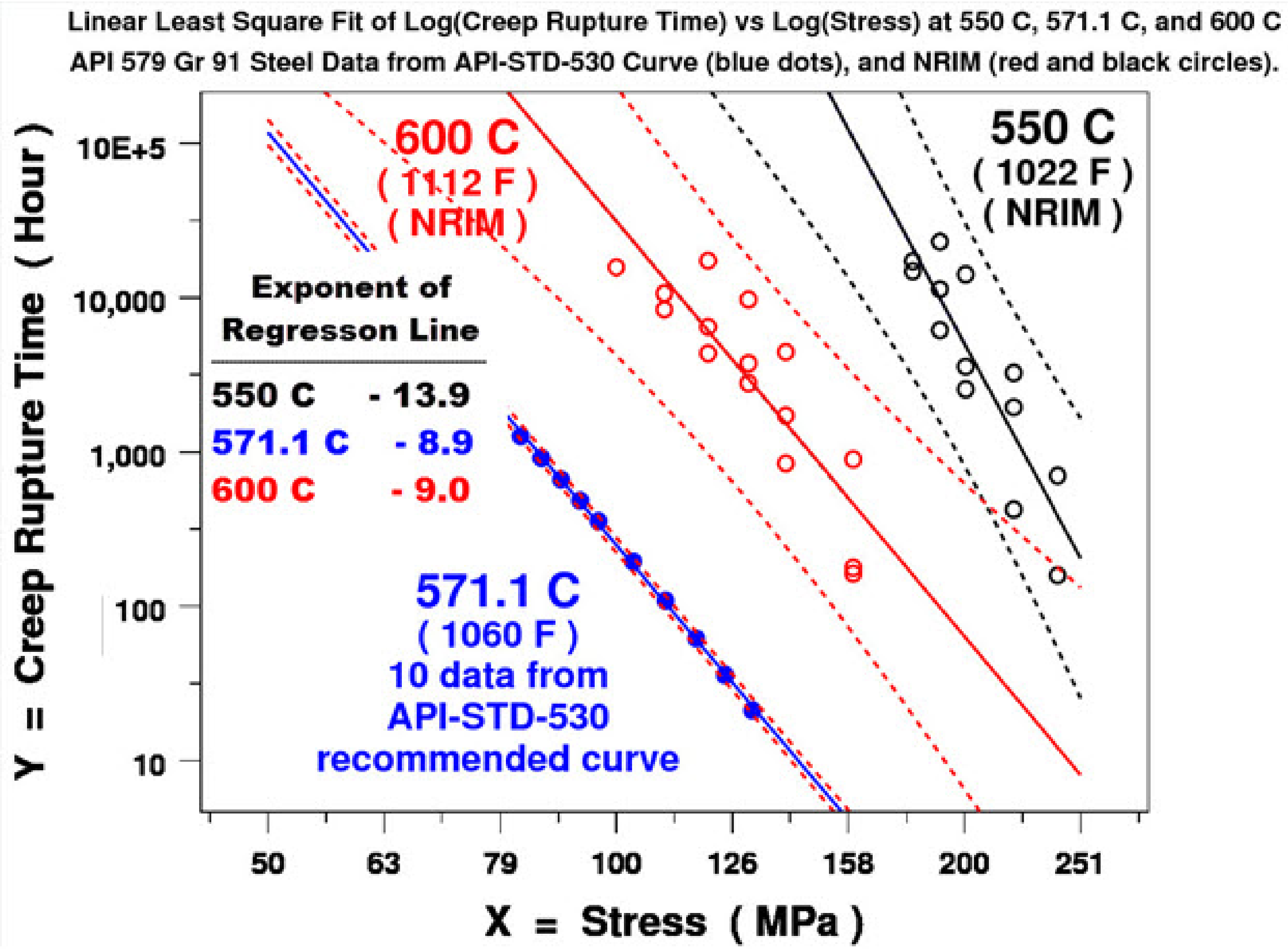
Email: fong@nist.gov

3.3 Linear Least SQuares (*LLSQ* - Regression).



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Tool-4

Repeatability and Reproducibility (*R&R*) Analysis and Variance Analysis (*ANOVA*).

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E691 INTERLAB (*R&R*)

Ref: "Standard Practice for Conducting an Interlaboratory Study to Determine the Precision of a test Method," ASTM International, West Conshohoceken, PA 19428-2959.

The purpose of the E691 INTERLAB command is to estimate the *precision* of a test method. Two important concepts in termining the precision are:

1. **Repeatability – repeatability concerns the variability between independent test results obtained within a single laboratory in the shortest practical period of time by a single operator with a specific set of test apparatus using test specimens taken at random from a single quantity of homogeneous material.**

2. **Reproducibility – reproducibility is the variability between single test results obtained in different laboratories, each of which has applied the test method to test specimens taken at random from a single quantity of homogeneous material.**

Analysis of Variance *(ANOVA)*

Description: Analysis of Variance (ANOVA) is a data analysis technique for examining the *significance* of the factors (= independent variables) in a multi-factor model.

The number of factors must be between 1 and 5 inclusive. Each factor then has a certain number of values it can have (referred to as the levels of a factor). The number of levels does not have to be the same for each factor.

Each factor and level combination is a cell (the number of cells is the product of the number of levels in each factor).

Balanced designs are those in which each cell has an equal number of observations and unbalanced designs are those in which the number of observations can vary between cells.

Tool-5

Design of Experiments

(*DEX*).

3. Six Tools of Eng. UQ

Tool-5 Design of Experiments (*DEX*).

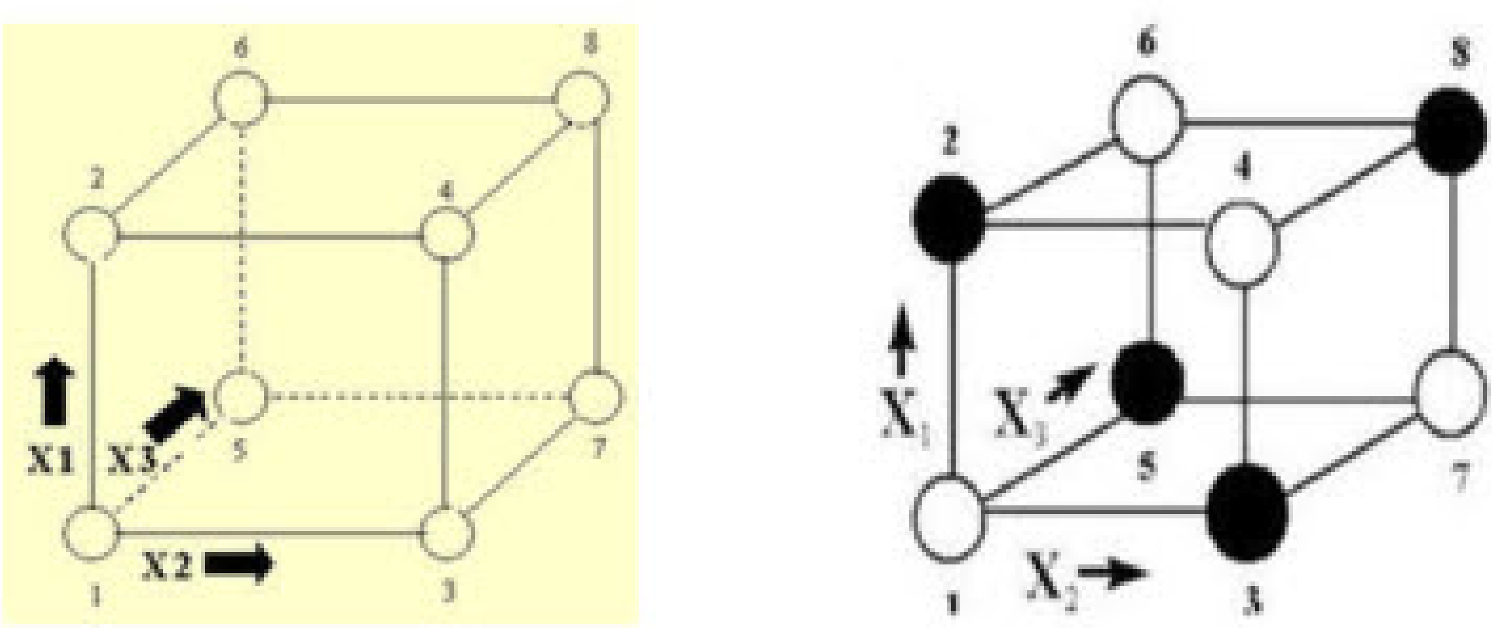
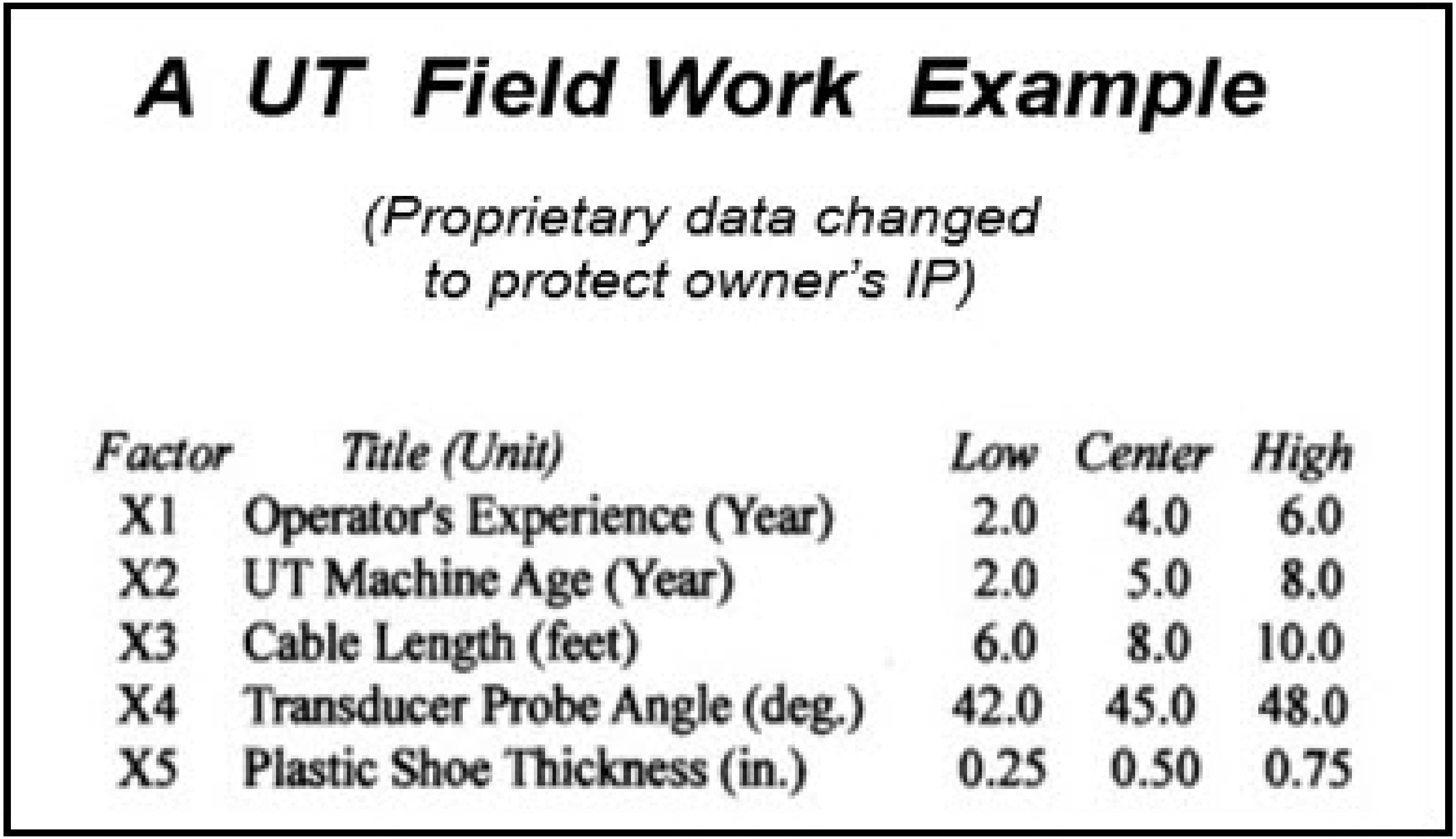


Figure 1. (left) **A full-factorial 8-run orthogonal design for 3 factors.**
(right) **A fractional factorial 4-run orthogonal design for 3 factors.**

3.5 Design of Experiments (DEX).



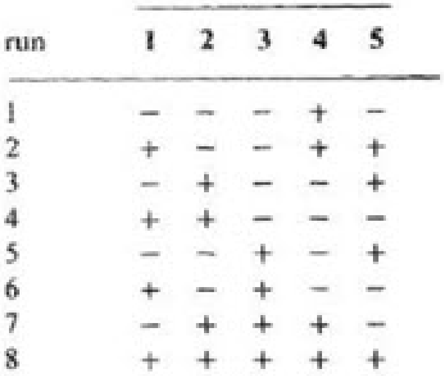
A UT Field Work Example

(Proprietary data changed to protect owner's IP)

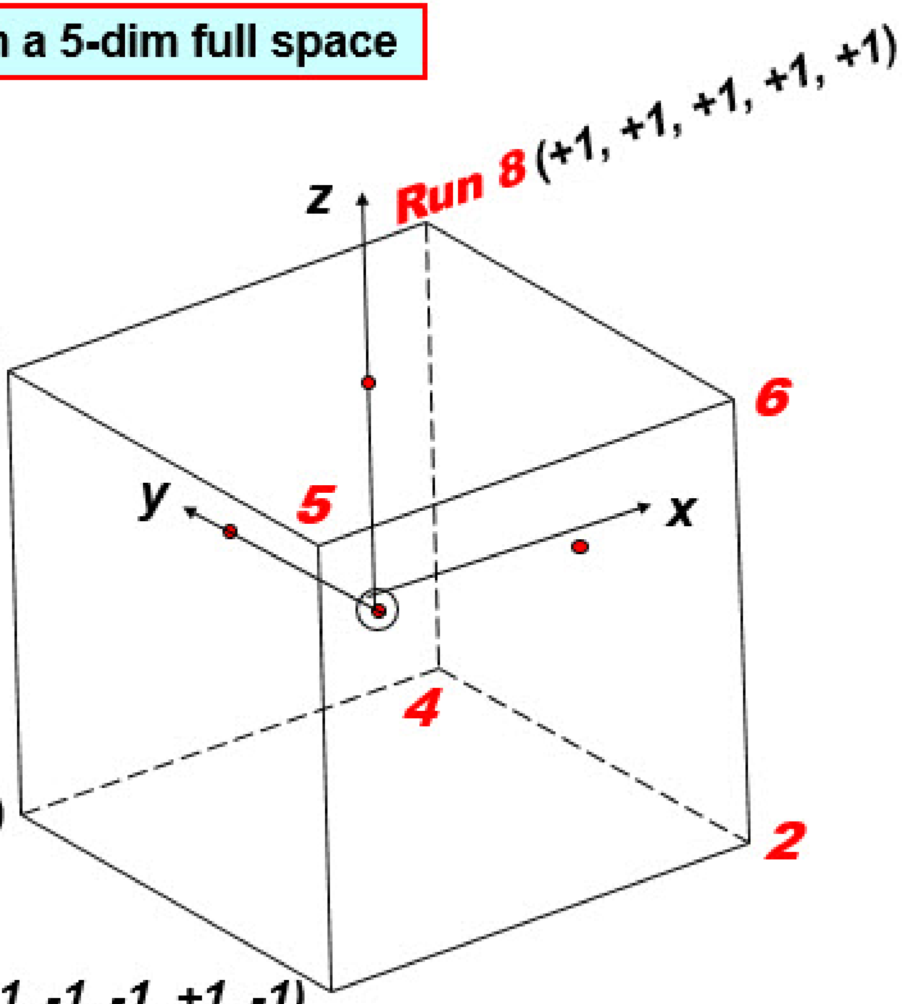
Factor	Title (Unit)	Low	Center	High
X1	Operator's Experience (Year)	2.0	4.0	6.0
X2	UT Machine Age (Year)	2.0	5.0	8.0
X3	Cable Length (feet)	6.0	8.0	10.0
X4	Transducer Probe Angle (deg.)	42.0	45.0	48.0
X5	Plastic Shoe Thickness (in.)	0.25	0.50	0.75

3.5 Design of Experiments (*DEX*).

Imagine a 3-dim subspace in a 5-dim full space



run	1	2	3	4	5
1	−	−	−	+	−
2	+	−	−	+	+
3	−	+	−	−	+
4	+	+	−	−	−
5	−	−	+	−	+
6	+	−	+	−	−
7	−	+	+	+	−
8	+	+	+	+	+



Tool-6

Non-Linear Least SQuares with Logistic Function

(*NLLSQ* - Lgs).

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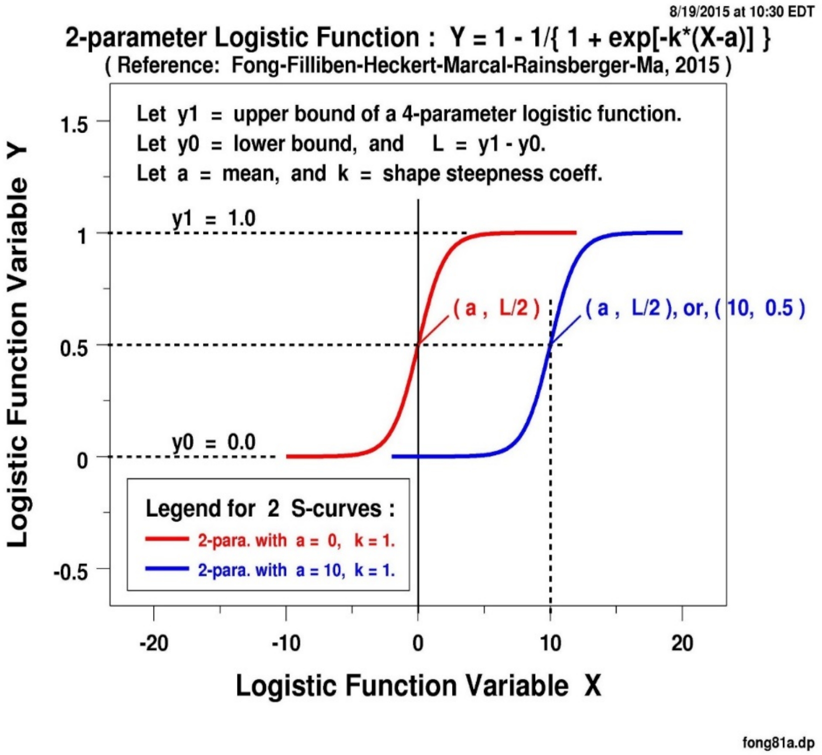
3. Six Tools of Eng. UQ

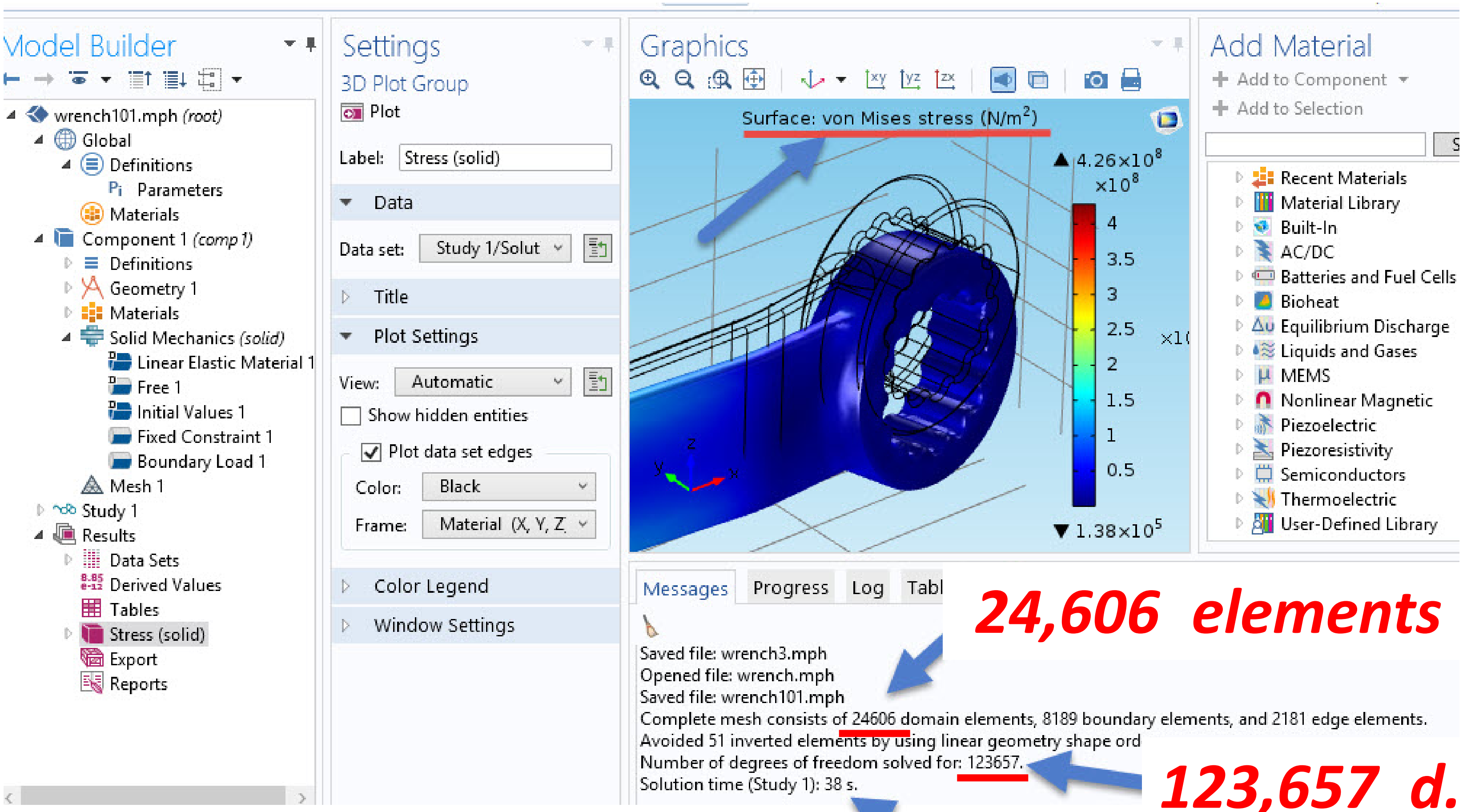
Tool-6 Non-Linear Least SQuares with Logistic Function (*NLLSQ* - Lgs).

$$f(\boldsymbol{x}) = y_1 - L * \{ \exp(-k*(\boldsymbol{x}-a)) / (1+\exp(-k*(\boldsymbol{x}-a))) \},$$

Pierre Francois Verhulst (1845)



Element Size: Fine



Label: Stress (solid)

Data set: Study 1/Solut

View: Automatic

☐ Show hidden entities

☑ Plot data set edges

Color: Black

Frame: Material (X, Y, Z

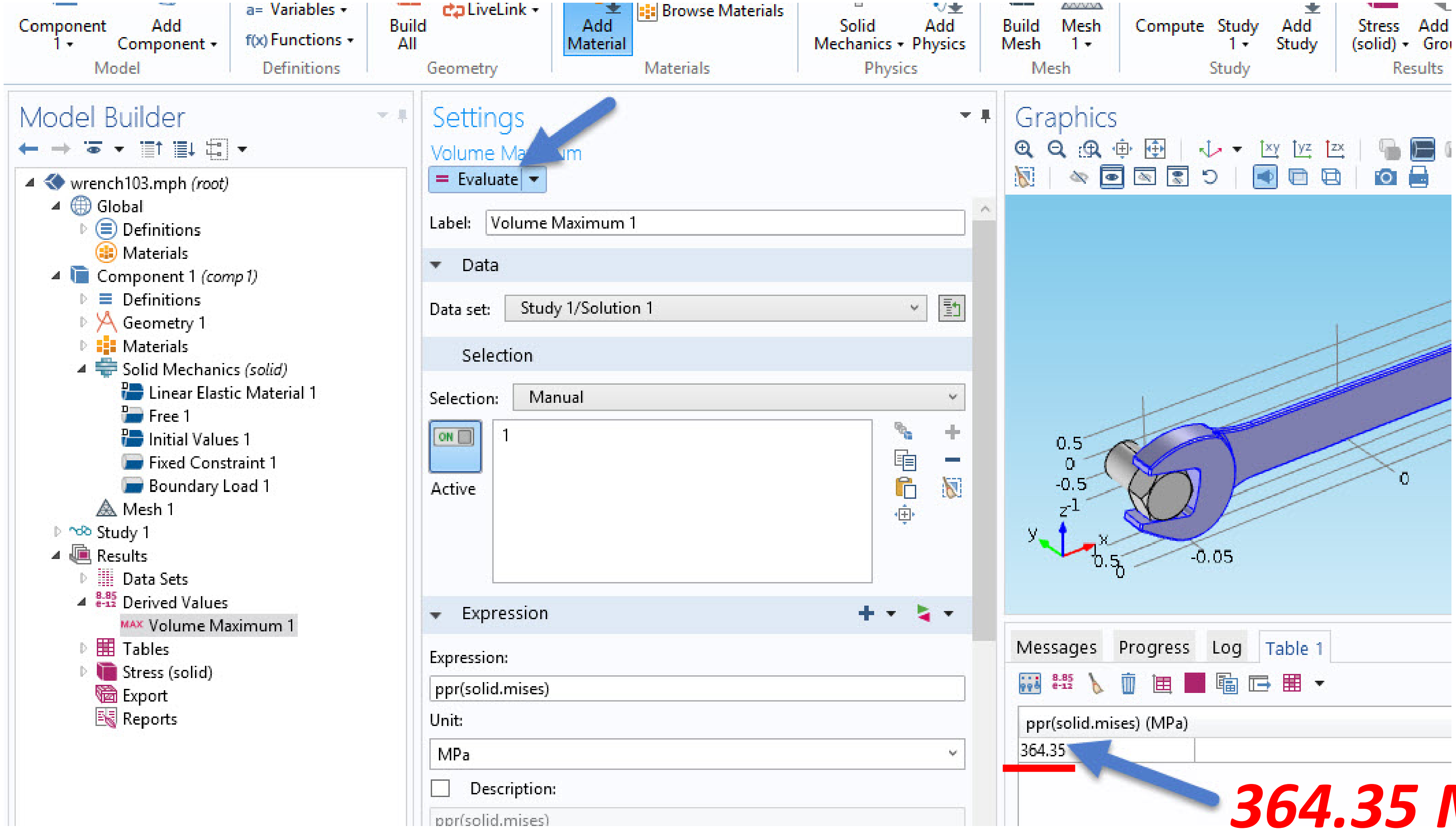
```
Saved file: wrench3.mph
Opened file: wrench.mph
Saved file: wrench101.mph
Complete mesh consists of 24606 domain elements, 8189 boundary elements, and 2181 edge elements.
Avoided 51 inverted elements by using linear geometry shape ord
Number of degrees of freedom solved for: 123657.
Solution time (Study 1): 38 s.
```

24,606 elements

123,657 d.o.f.

3.6 Non-Linear Least Square Fit with a Logistic Function (NLLSQ)

Element Size: Fine



Label: Volume Maximum 1

Data set: Study 1/Solution 1

Selection: Manual

1

Expression: ppr(solid.mises)

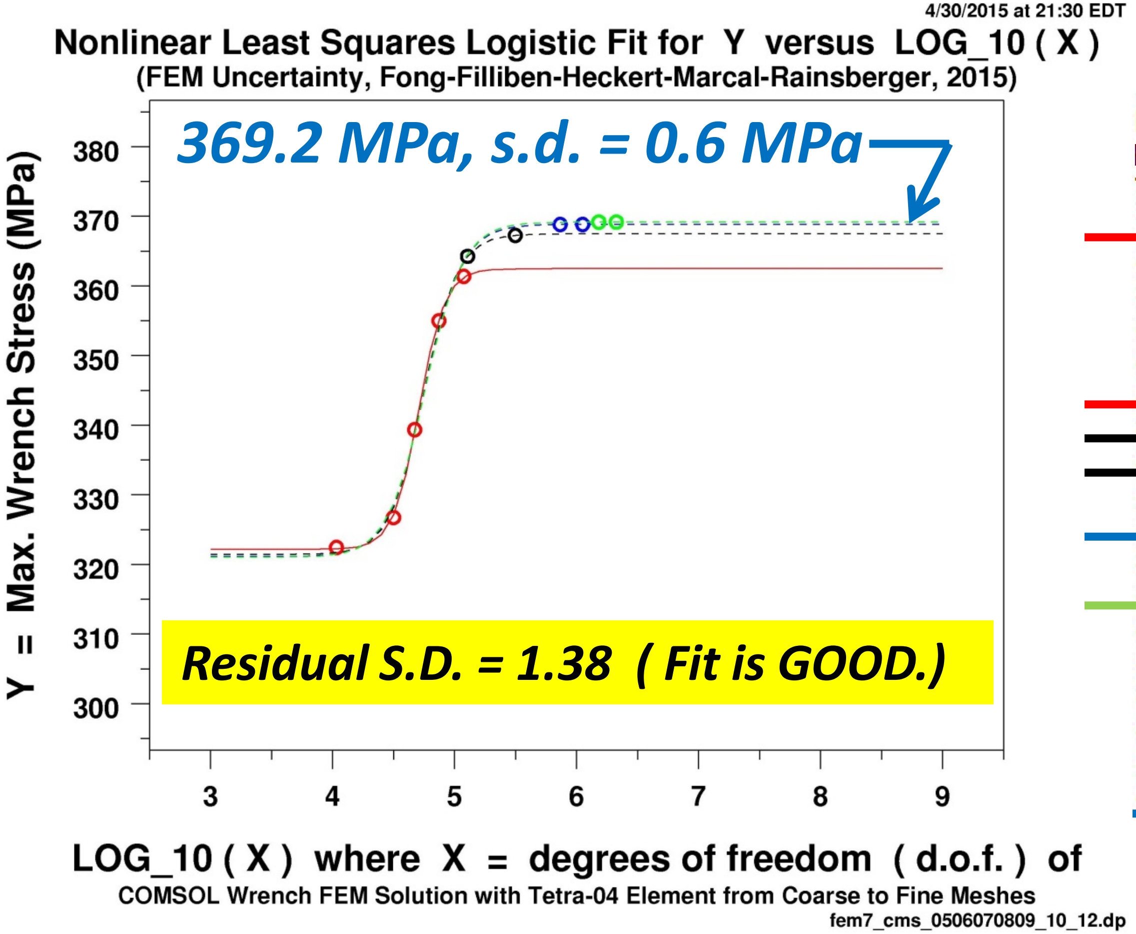
Unit: MPa

☐ Description: ppr(solid.mises)

ppr(solid.mises) (MPa)
364.35

364.35 MPa

3.6 Non-Linear Least Square Fit with a Logistic Function (NLLSQ)



Nonlinear Least Squares Logistic Fit for Y versus LOG_10 (X)

(FEM Uncertainty, Fong-Filliben-Heckert-Marcal-Rainsberger, 2015)

369.2 MPa, s.d. = 0.6 MPa

Residual S.D. = 1.38 (Fit is GOOD.)

LOG_10 (X) where X = degrees of freedom (d.o.f.) of

COMSOL Wrench FEM Solution with Tetra-04 Element from Coarse to Fine Meshes

FEM Result Y (MPa)	Degrees of Freedom (dof) X
322.45	10743
326.76	31476
339.37	47022
355.02	74226
361.40	118750
364.35	127663
368.55	313970
369.24	732220
369.72	1119600
369.73	1517800
369.61	2113600
369.54	2670100
369.80	3411800
369.72	4193000
369.72	5033200
369.61	5919600
369.71	6,932,883

Ans. Max. Mises Stress at 95 % confidence level = (368.0, ... , 370.4 MPa)

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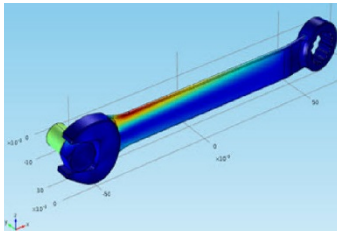
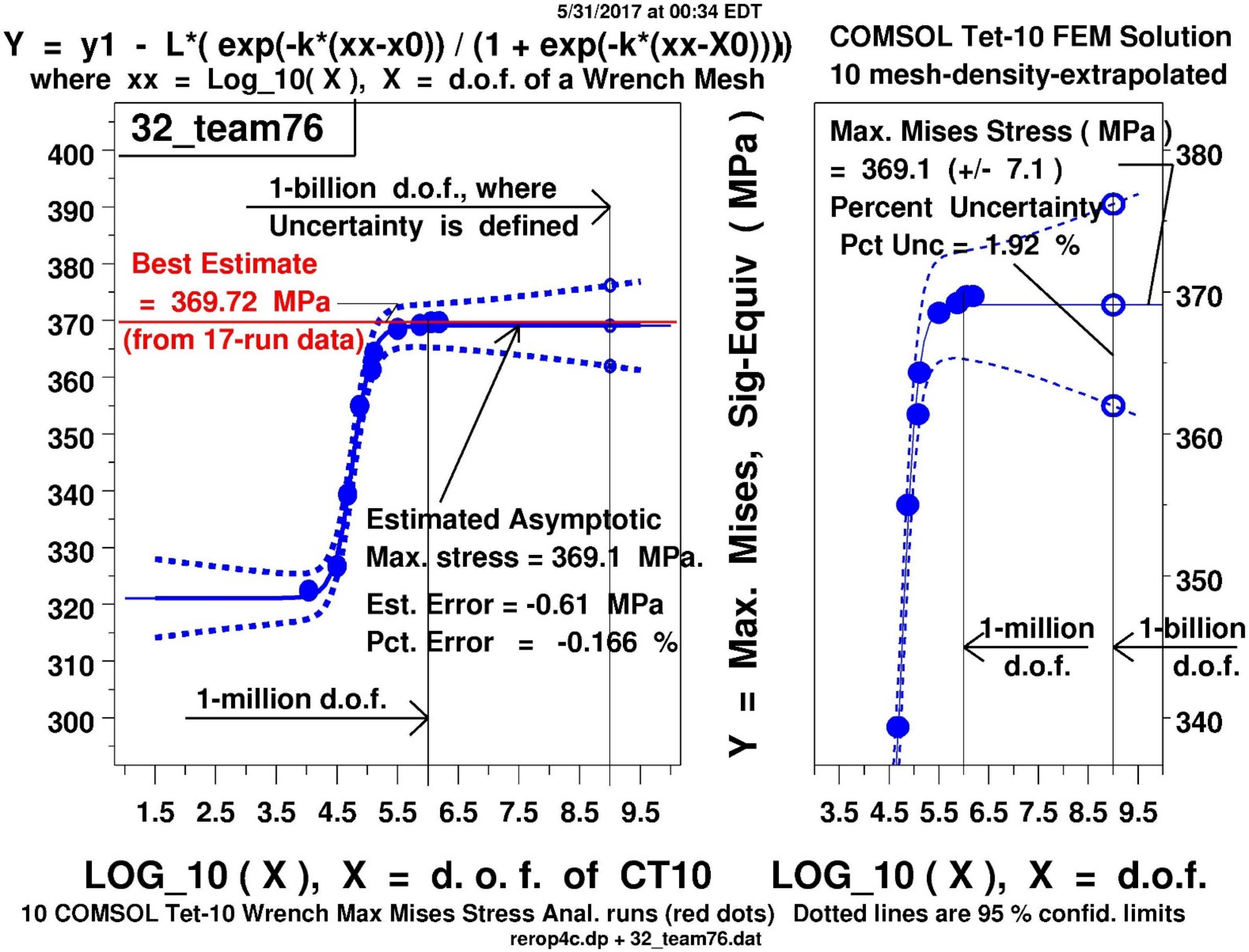
National Institute of Standards and Technology

3.6 Non-Linear Least Square Fit with a Logistic Function (NLLSQ)





Summary of Six Tools of UQ

		New Concept ?	New Software ?
3.1	*GoF / 64*	No.	*Dataplot*
3.2	coni , Predi , *LTL*	No.	*Dataplot, R*
3.3	*LLSQ* - Regression	No.	*Dataplot, R*
3.5	*R&R + ANOVA*	No.	*Dataplot, R*
3.5	*DEX*	*Yes.*	*Dataplot, R*
3.6	*NLLSQ*	*Yes.*	*Dataplot*

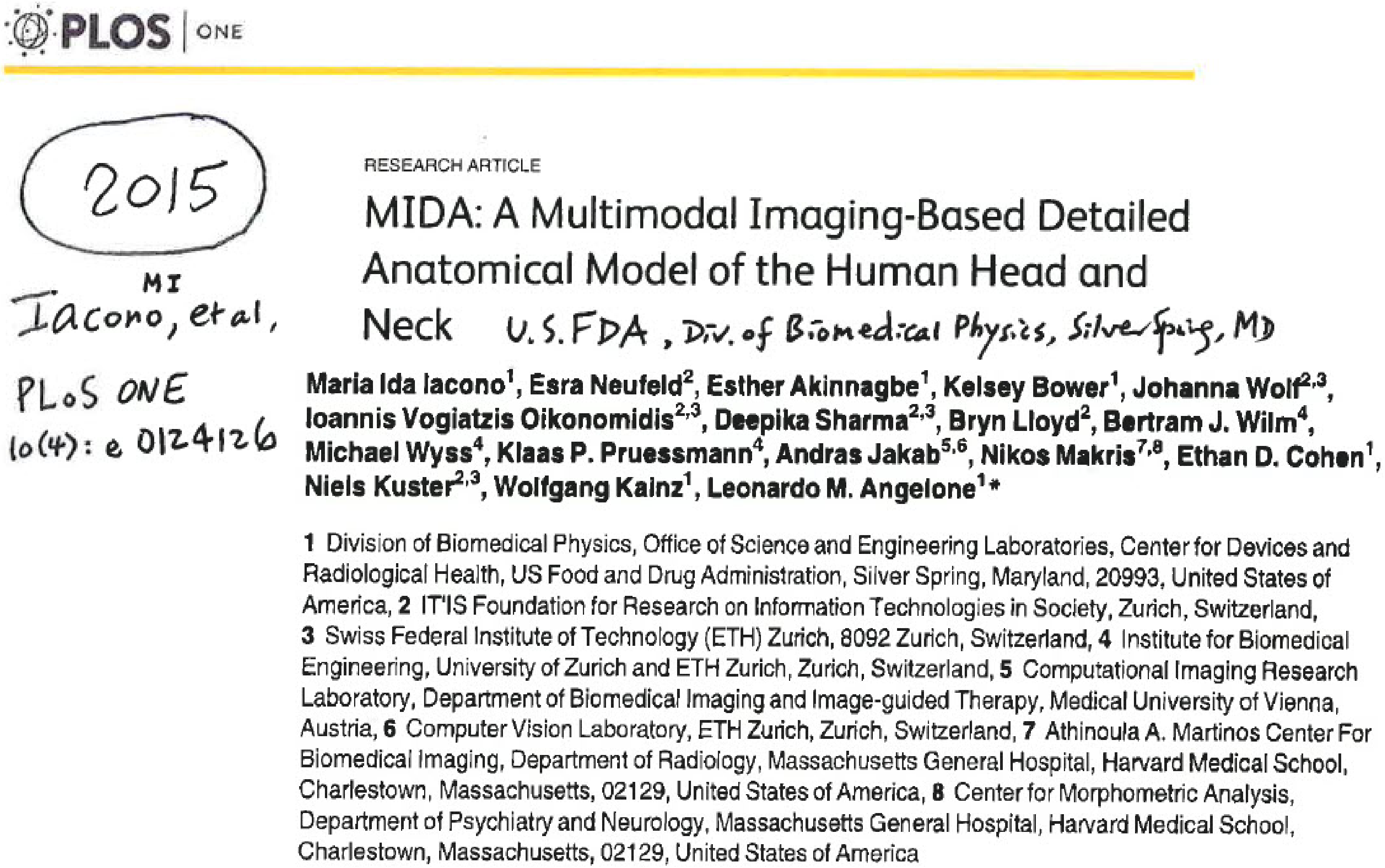
No. of Slides	*Subtotal*
28	*48*

4. UQ Tool-2, 3, 4, & 5 :

Brain Metrology

Research.

4. Tool-2, 3, 4, 5 : Brain Metrology Research



PLOS | ONE

2015

MI Iacono, et al,

PLoS ONE 10(4): e 0124126

RESEARCH ARTICLE

MIDA: A Multimodal Imaging-Based Detailed Anatomical Model of the Human Head and Neck

U.S. FDA, Div. of Biomedical Physics, Silver Spring, MD

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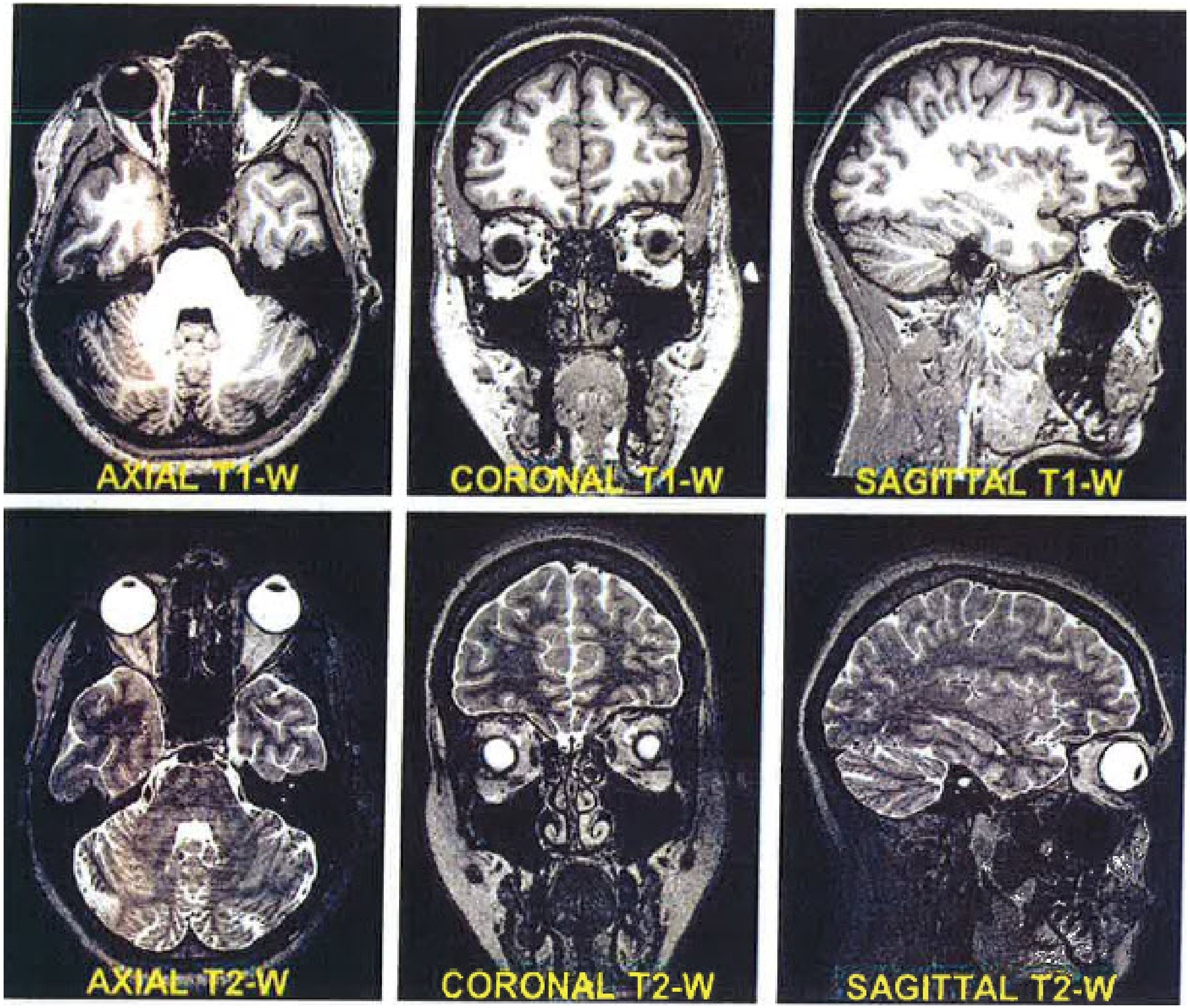
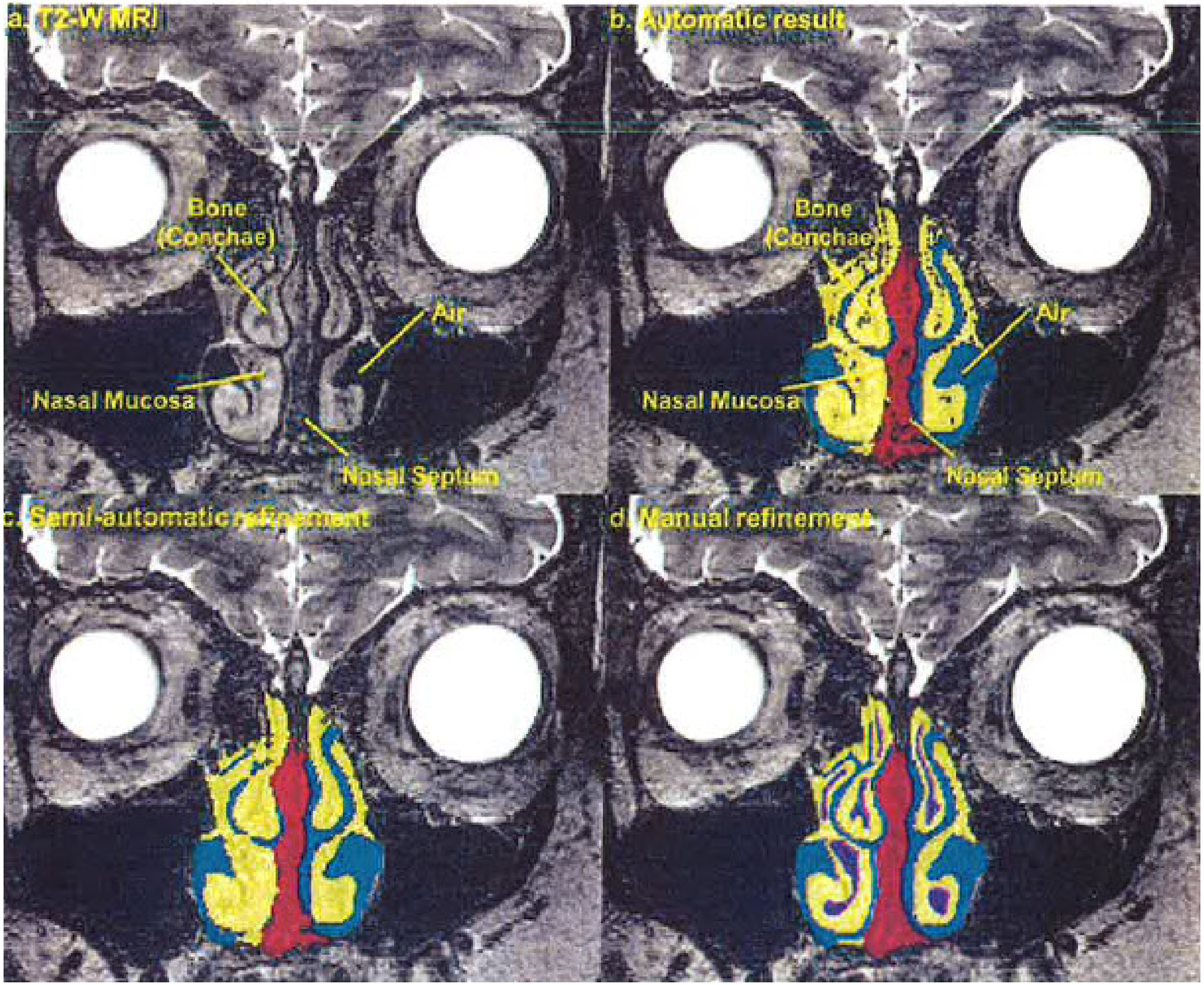
1 Division of Biomedical Physics, Office of Science and Engineering Laboratories, Center for Devices and Radiological Health, US Food and Drug Administration, Silver Spring, Maryland, 20993, United States of America, **2** IT'IS Foundation for Research on Information Technologies in Society, Zurich, Switzerland, **3** Swiss Federal Institute of Technology (ETH) Zurich, 8092 Zurich, Switzerland, **4** Institute for Biomedical Engineering, University of Zurich and ETH Zurich, Zurich, Switzerland, **5** Computational Imaging Research Laboratory, Department of Biomedical Imaging and Image-guided Therapy, Medical University of Vienna, Austria, **6** Computer Vision Laboratory, ETH Zurich, Zurich, Switzerland, **7** Athinoula A. Martinos Center For Biomedical Imaging, Department of Radiology, Massachusetts General Hospital, Harvard Medical School, Charlestown, Massachusetts, 02129, United States of America, **8** Center for Morphometric Analysis, Department of Psychiatry and Neurology, Massachusetts General Hospital, Harvard Medical School, Charlestown, Massachusetts, 02129, United States of America

4. Tool-2, 3, 4, 5 : Brain Metrology Research



4. Tool-2, 3, 4, 5 : Brain Metrology Research



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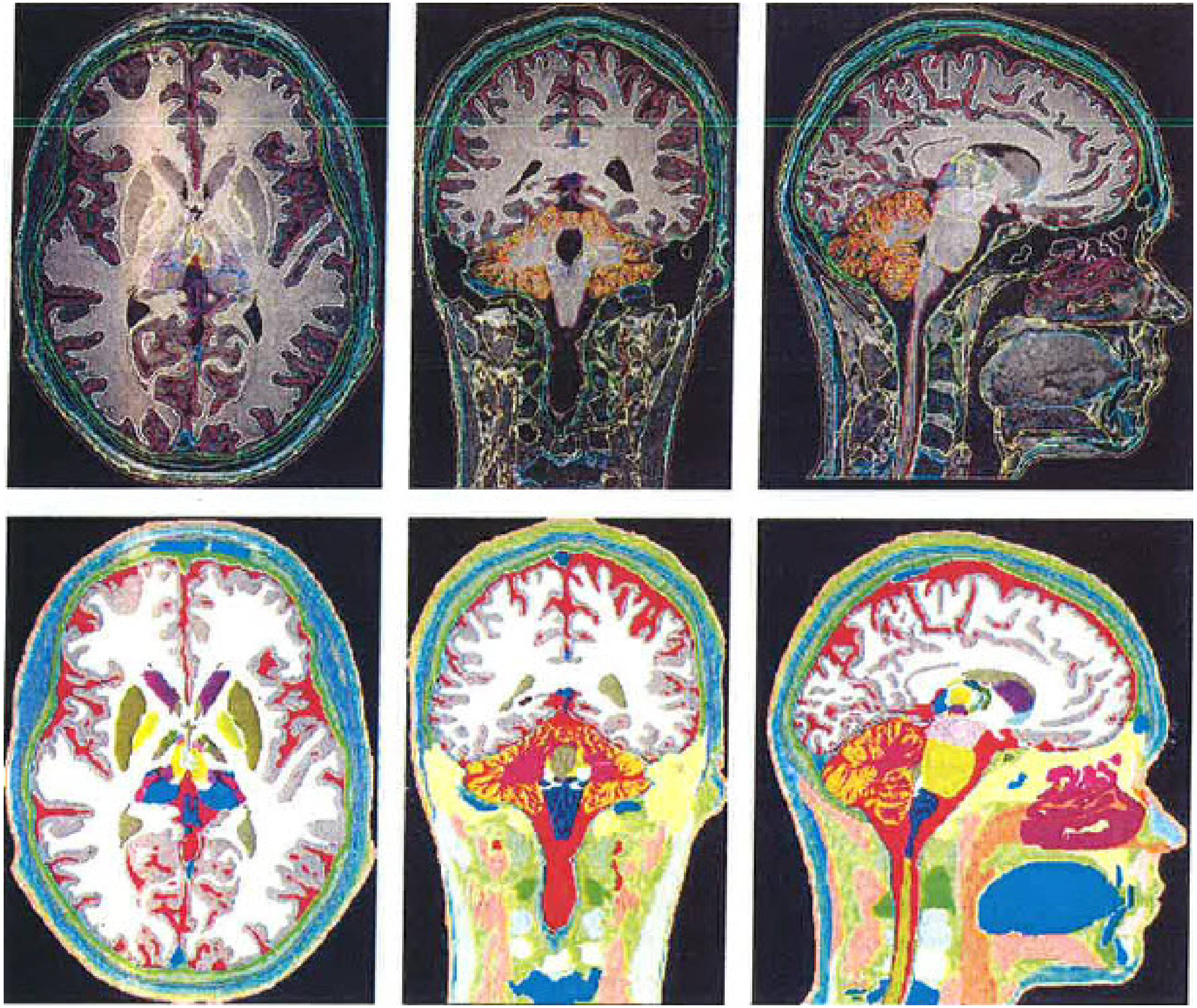
4. Tool-2, 3, 4, 5 : Brain Metrology Research



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4. Tool-2, 3, 4, 5 : Brain Metrology Research

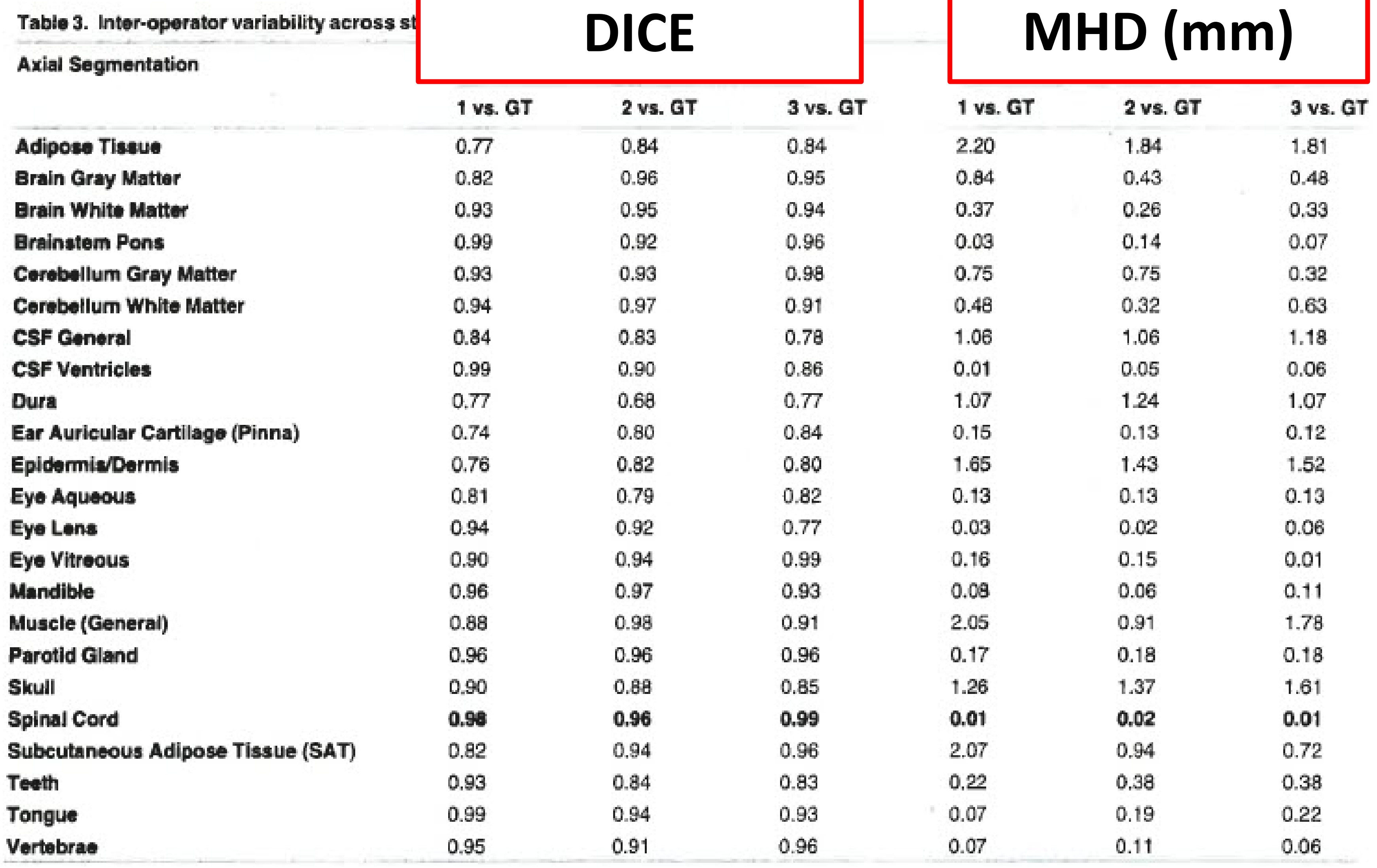


Table 3. Inter-operator variability across st

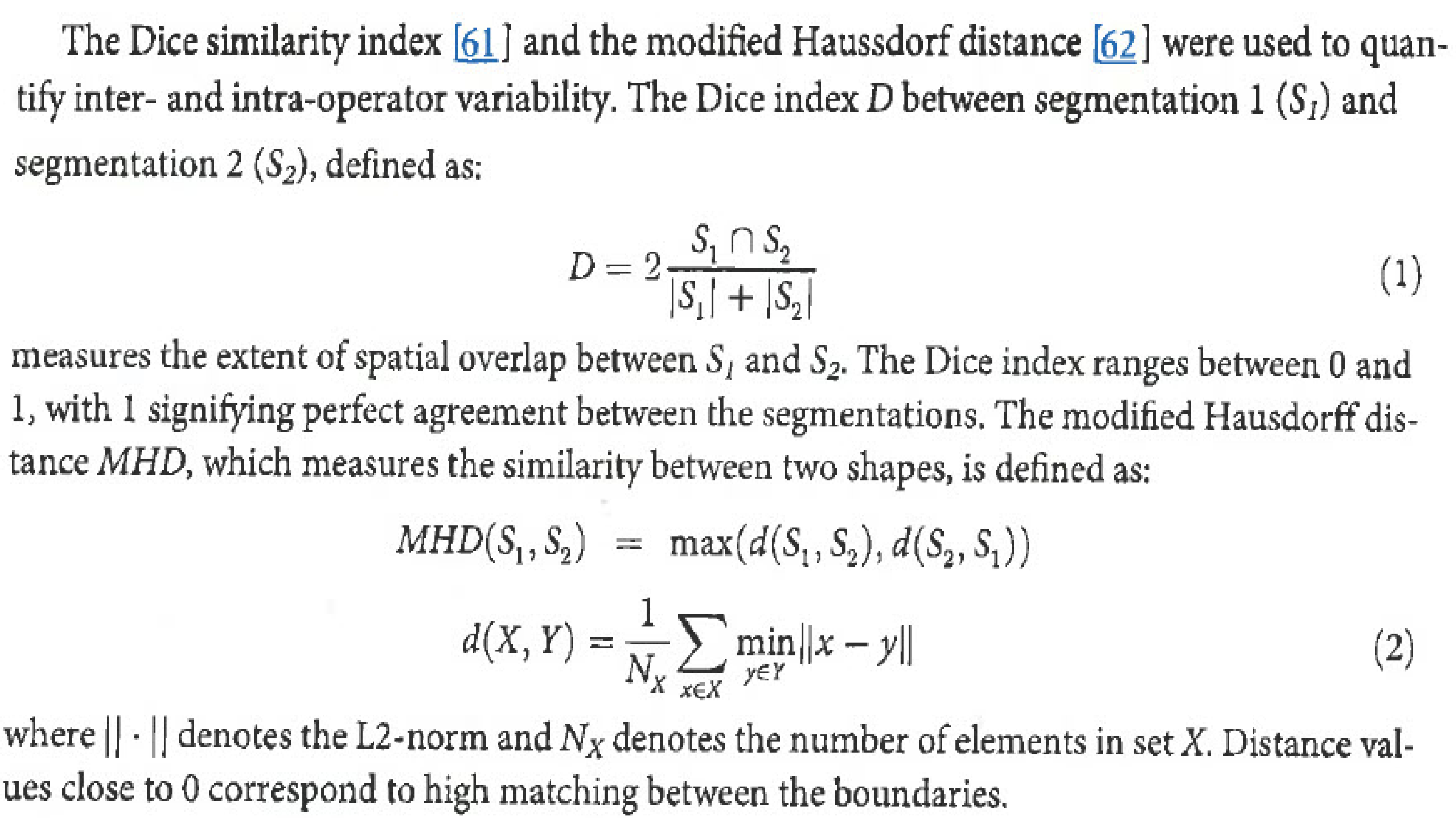
Axial Segmentation

	DICE			MHD (mm)		
	1 vs. GT	2 vs. GT	3 vs. GT	1 vs. GT	2 vs. GT	3 vs. GT
Adipose Tissue	0.77	0.84	0.84	2.20	1.84	1.81
Brain Gray Matter	0.82	0.96	0.95	0.84	0.43	0.48
Brain White Matter	0.93	0.95	0.94	0.37	0.26	0.33
Brainstem Pons	0.99	0.92	0.96	0.03	0.14	0.07
Cerebellum Gray Matter	0.93	0.93	0.98	0.75	0.75	0.32
Cerebellum White Matter	0.94	0.97	0.91	0.48	0.32	0.63
CSF General	0.84	0.83	0.78	1.06	1.06	1.18
CSF Ventricles	0.99	0.90	0.86	0.01	0.05	0.06
Dura	0.77	0.68	0.77	1.07	1.24	1.07
Ear Auricular Cartilage (Pinna)	0.74	0.80	0.84	0.15	0.13	0.12
Epidermis/Dermis	0.76	0.82	0.80	1.65	1.43	1.52
Eye Aqueous	0.81	0.79	0.82	0.13	0.13	0.13
Eye Lens	0.94	0.92	0.77	0.03	0.02	0.06
Eye Vitreous	0.90	0.94	0.99	0.16	0.15	0.01
Mandible	0.96	0.97	0.93	0.08	0.06	0.11
Muscle (General)	0.88	0.98	0.91	2.05	0.91	1.78
Parotid Gland	0.96	0.96	0.96	0.17	0.18	0.18
Skull	0.90	0.88	0.85	1.26	1.37	1.61
Spinal Cord	**0.98**	**0.96**	**0.99**	**0.01**	**0.02**	**0.01**
Subcutaneous Adipose Tissue (SAT)	0.82	0.94	0.96	2.07	0.94	0.72
Teeth	0.93	0.84	0.83	0.22	0.38	0.38
Tongue	0.99	0.94	0.93	0.07	0.19	0.22
Vertebrae	0.95	0.91	0.96	0.07	0.11	0.06

4. Tool-2, 3, 4, 5 : Brain Metrology Research

DICE

MHD (mm)



The Dice similarity index [61] and the modified Haussdorf distance [62] were used to quantify inter- and intra-operator variability. The Dice index D between segmentation 1 (S_1) and segmentation 2 (S_2), defined as:

$$D = 2\frac{S_1 \cap S_2}{|S_1| + |S_2|} \qquad (1)$$

measures the extent of spatial overlap between S_1 and S_2. The Dice index ranges between 0 and 1, with 1 signifying perfect agreement between the segmentations. The modified Hausdorff distance MHD, which measures the similarity between two shapes, is defined as:

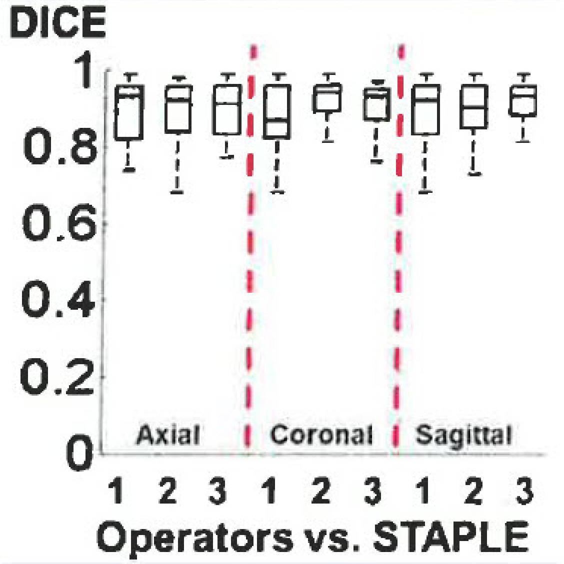
$$MHD(S_1, S_2) = \max(d(S_1, S_2), d(S_2, S_1))$$

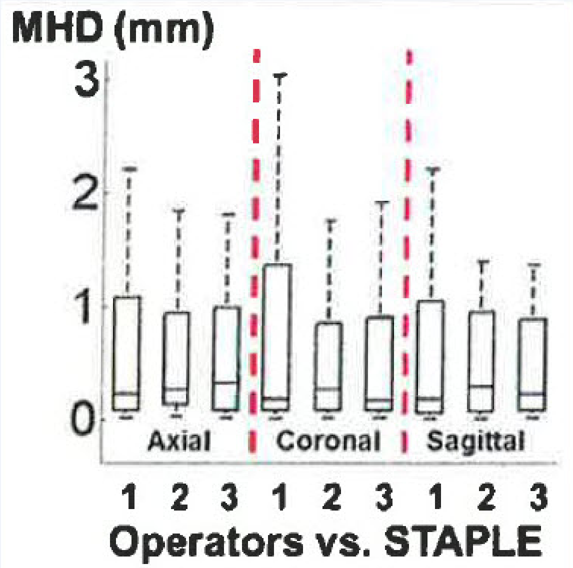
$$d(X, Y) = \frac{1}{N_X}\sum_{x \in X} \min_{y \in Y} \|x - y\| \qquad (2)$$

where $\|\cdot\|$ denotes the L2-norm and N_X denotes the number of elements in set X. Distance values close to 0 correspond to high matching between the boundaries.

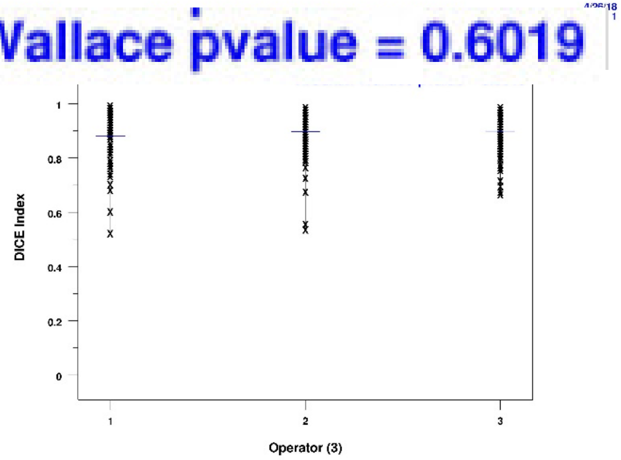
4. Tool-2, 3, 4, 5 : Brain Metrology Research

2015 Data
36 Tissues
of which
15 triples
7 doubles
14 Singles
Unbalanced



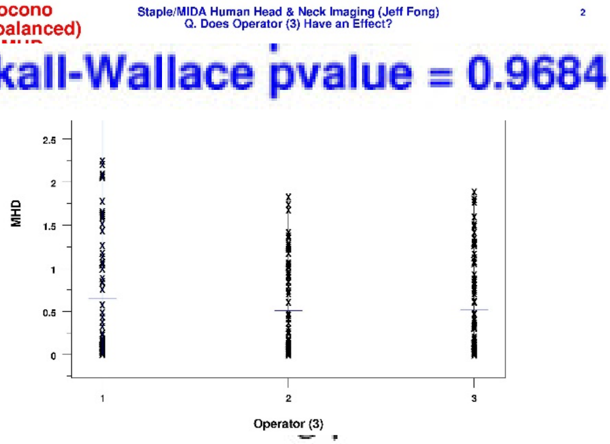


Kruskall-Wallace pvalue = 0.6019



Iocono (Unbalanced)

Kruskall-Wallace pvalue = 0.9684

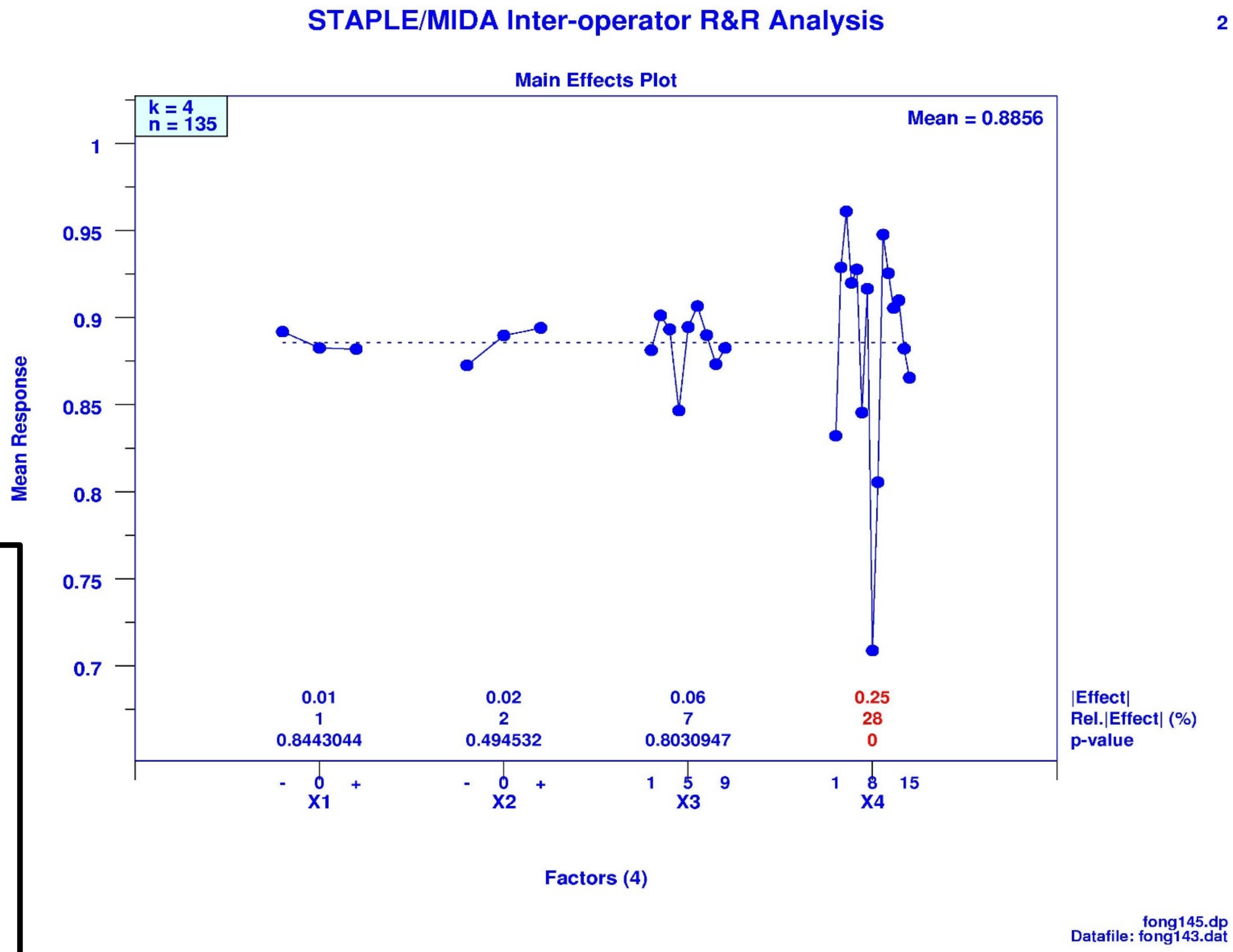


non-parametric Kruskal-Wallis test of variance was used for groups' comparison. The analysis suggested no significant inter-observer variability among operators in terms of *D* indexes (p-value = 0.61) and *MHD* values (p-value = 0.96) with 95% confidence.

4. Tool-2, 3, 4, 5 : Brain Metrology Research

2015 Data
36 Tissues
of which
15 triples
7 doubles
14 Singles
Unbalanced

2015 subset
15 Tissues
each of
which is a
triple (3-dir)
balanced

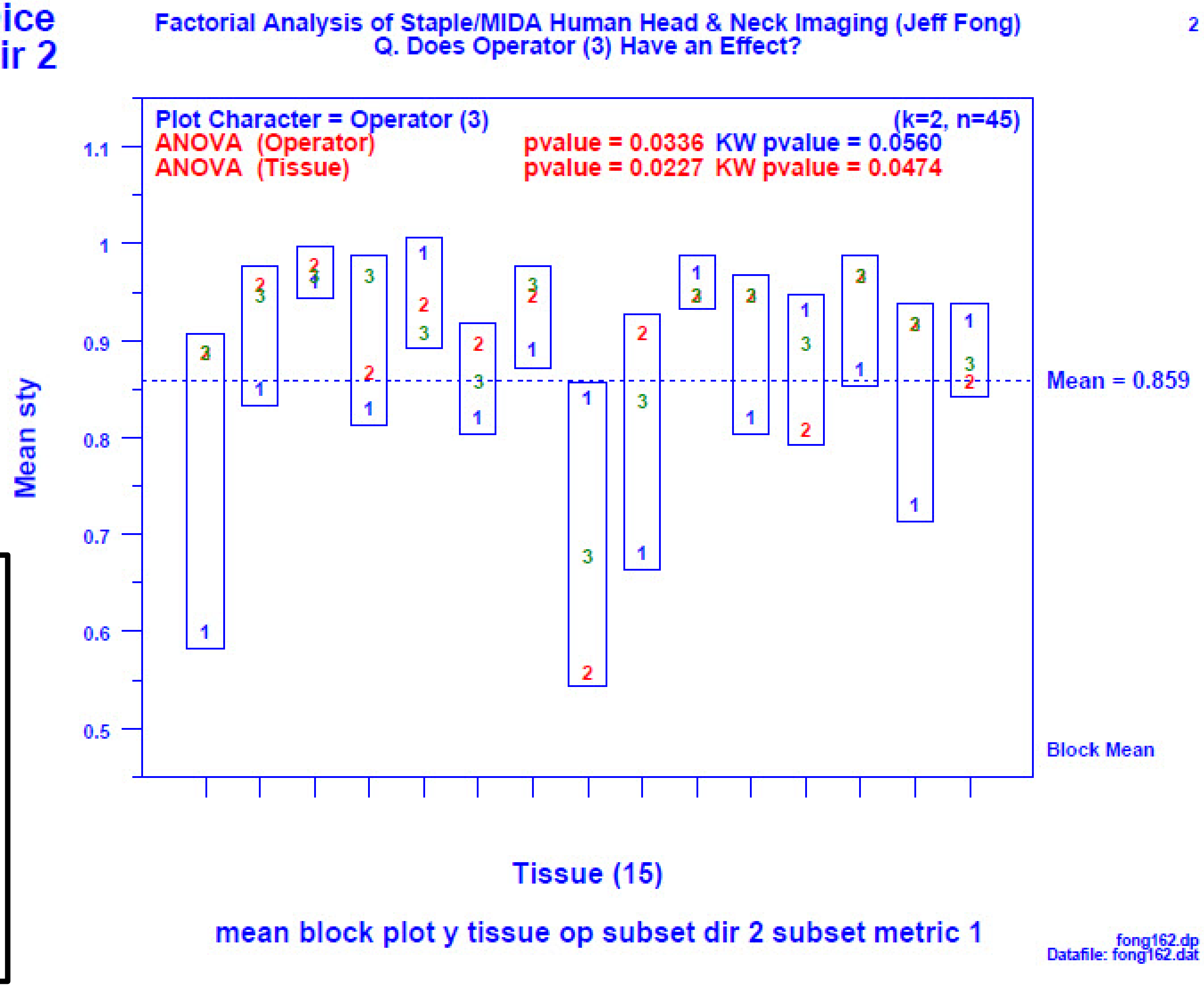


4. Tool-2, 3, 4, 5 : Brain Metrology Research

2015 Data
36 Tissues
of which
15 triples
7 doubles
14 Singles
Unbalanced

2015 subset
15 Tissues
each of
which is a
triple (3-dir)
balanced

Dice
Dir 2

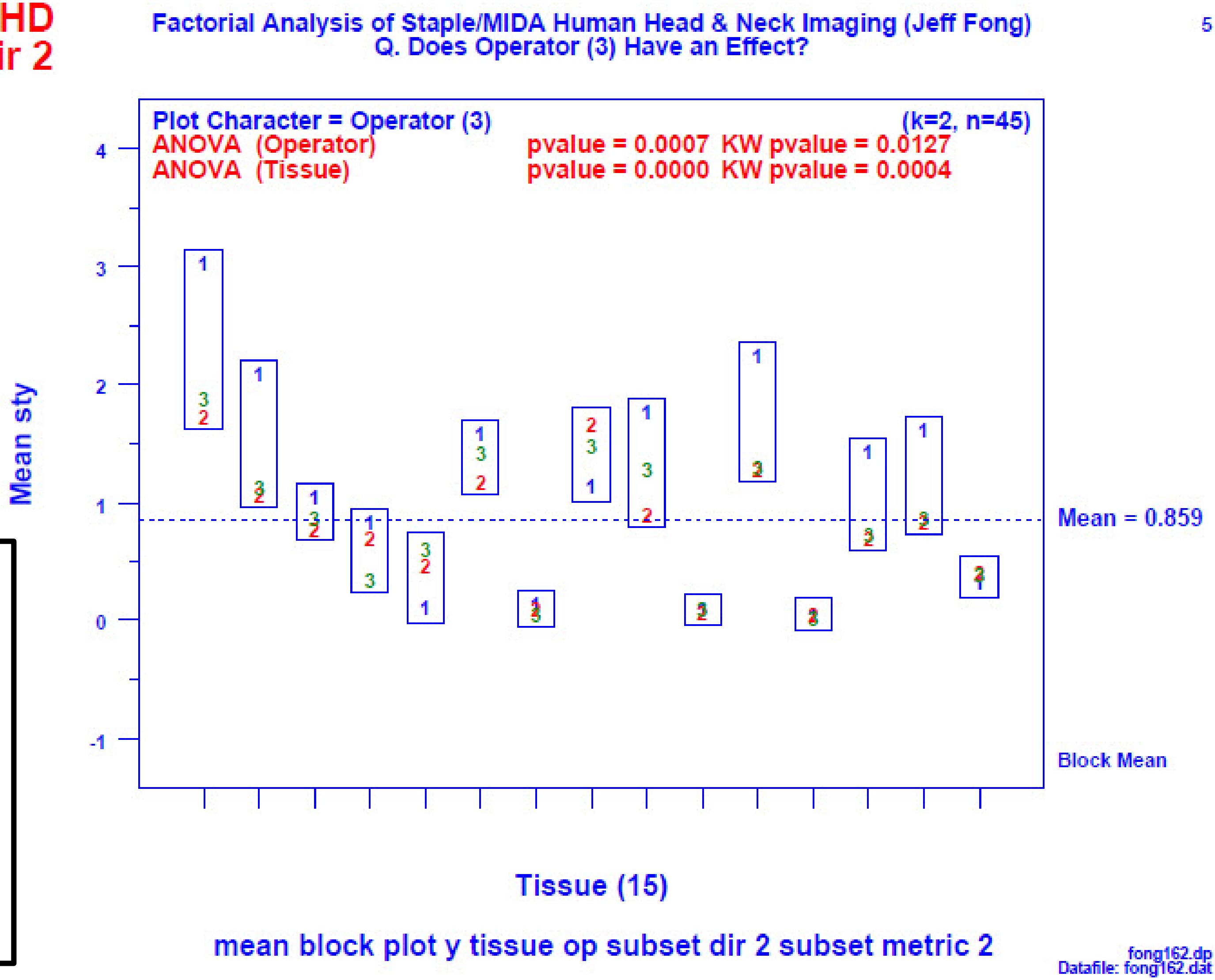


4. Tool-2, 3, 4, 5 : Brain Metrology Research

2015 Data
36 Tissues
of which
15 triples
7 doubles
14 Singles
Unbalanced

2015 subset
15 Tissues
each of
which is a
triple (3-dir)
balanced

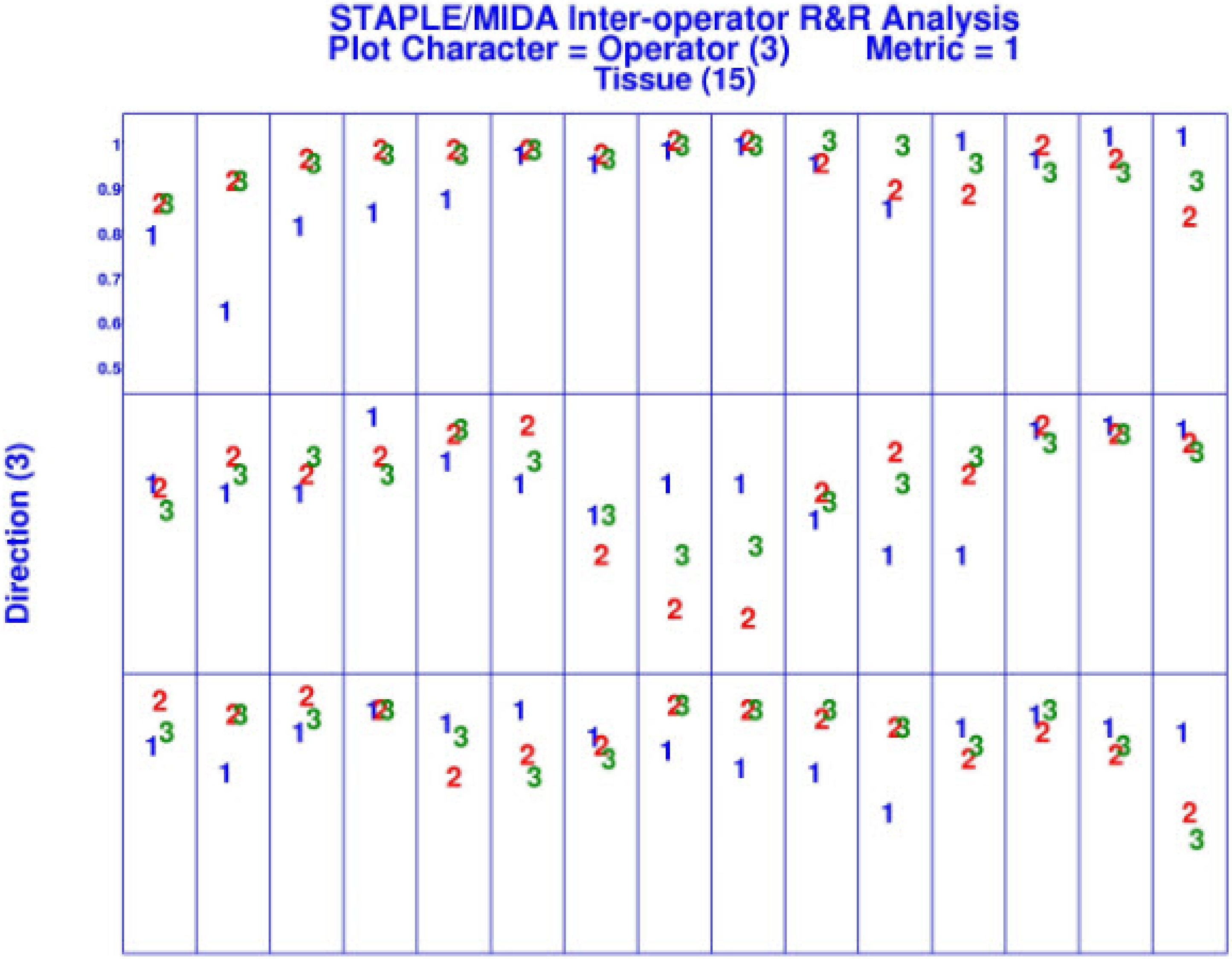
MHD
Dir 2



4. Tool-2, 3, 4, 5 : Brain Metrology Research

2015 subset

15 Tissues each of which is a triple (3-dir) **balanced**



No. of Slides	*Subtotal*
11	*59*

No. of slides	*Subtotal*
1	*60*

8. Concluding Remarks

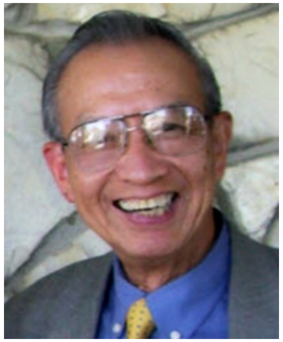
8.1 Uncertainty in all aspects of engineering from design to manufacturing, testing, operation, maintenance, and life extension, is quantifiable with the help of modern computing.

8.2 Uncertainty quantification (UQ), however, does not come without cost, and engineers need to learn how to perform an Elementary Probability Risk Analysis (EPRA) to justify the cost of UQ against the benefit of uncertainty-based asset management that reduces failure probability to an "acceptable" minimum.

Disclaimer

Certain commercial equipment, instruments, materials, or computer software are identified in this talk in order to specify the experimental or computational procedure adequately. Such identification is not intended to imply recommendation or endorsement by the National Institute of Standards & Technology, nor is it intended to imply that the materials, equipment, or software identified are necessarily the best available for the purpose.

Speaker's Biographical Sketch



Dr. Jeffrey T. Fong has been Physicist and Project Manager at the Applied and Computational Mathematics Division, Information Technology Laboratory, National Institute of Standards and Technology (NIST), **Gaithersburg, MD, since 1966.**

He was educated at the University of Hong Kong (B.Sc., Engineering, first class honors, 1955), Columbia University (M.S., Engineering Mechanics, 1961), and Stanford (Ph.D., Applied Mechanics and Mathematics, 1966). Prior to 1966, he worked as a design engineer (1955-63) on numerous power plants (hydro, fossil-fuel, nuclear) at Ebasco Services, Inc., in New York City, and as teaching & research assistant (1963-66) on engineering mechanics at Stanford University.

During his 40+ years at NIST**, he has conducted research, provided consulting services, and taught numerous short courses on mathematical and computational modeling with uncertainty estimation for fatigue, fracture, high-temperature creep, nondestructive evaluation, electromagnetic behavior, and failure analysis of a broad range of materials ranging from paper, ceramics, glass, to polymers, composites, metals, semiconductors, and biological tissues.**

A licensed professional engineer (P.E.) in the State of New York since 1962 and a chartered civil engineer in the United Kingdom and British Commonwealth (A.M.I.C.E.) since 1968, he has authored or co-authored more than 100 technical papers, and edited or co-edited 17 national or international conference proceedings. He was elected Fellow of ASTM in 1982 and Fellow of ASME in 1984. In 1993, he was awarded the prestigious ASME *Pressure Vessels and Piping Medal.* Most recently, he was honored at the 2014 International Conference on Computational & Experimental Engineering & Sciences (ICCES) with a *Lifetime Achievement Medal.*

Since 2006, he has been Adjunct Professor of Mechanical Engineering and Mechanics at Drexel University **and taught a graduate-level 3-credit course on "Finite Element Method Uncertainty Analysis." Since Jan. 2010, he has given every 6 months an on-line 3-hour short course at** Stanford University **on "Reliability and Uncertainty Estimation of FEM Models of Composite Structures." In 2012, he was appointed Adjunct Professor of Nuclear and Risk Engineering at the** City University of Hong Kong**, and Distinguished Guest Professor at the** East China University of Science & Technology**, Shanghai, China, to teach annually a 1-credit 16-hour short course on "Engineering Reliability and Risk Analysis."**

Appendix (5 + 6 + 7 = 24 more slides)

1. Why is UQ important in Engineering ?
2. Example of an *Easy-to-use* UQ Tool for Engineers.
3. Six *Easy-to-use* (*ETU*) Tools of Engineering UQ .
4. Tool-2, 3, 4, 5. UQ for Brain Metrology Research.
5. Tool-3, & 5. UQ for Flaw Detection and Sizing.
6. Tool-1 & 2. UQ for Design of an Aircraft Window.
7. Tool-6. UQ for Maintenance Decision Making.
8. Concluding Remarks.

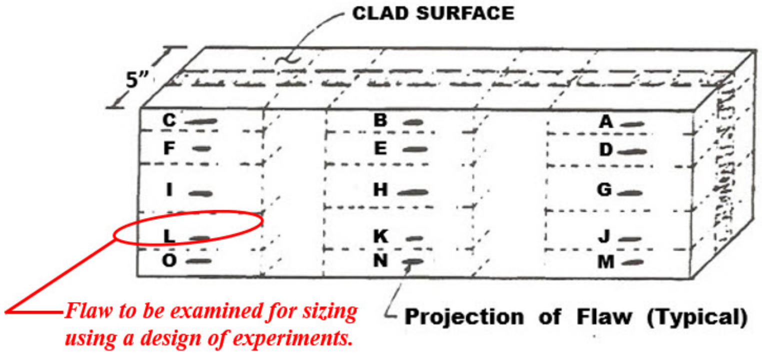
5. UQ Tool-3, and 5 :

Flaw Detection and

Sizing (NDE UQ).

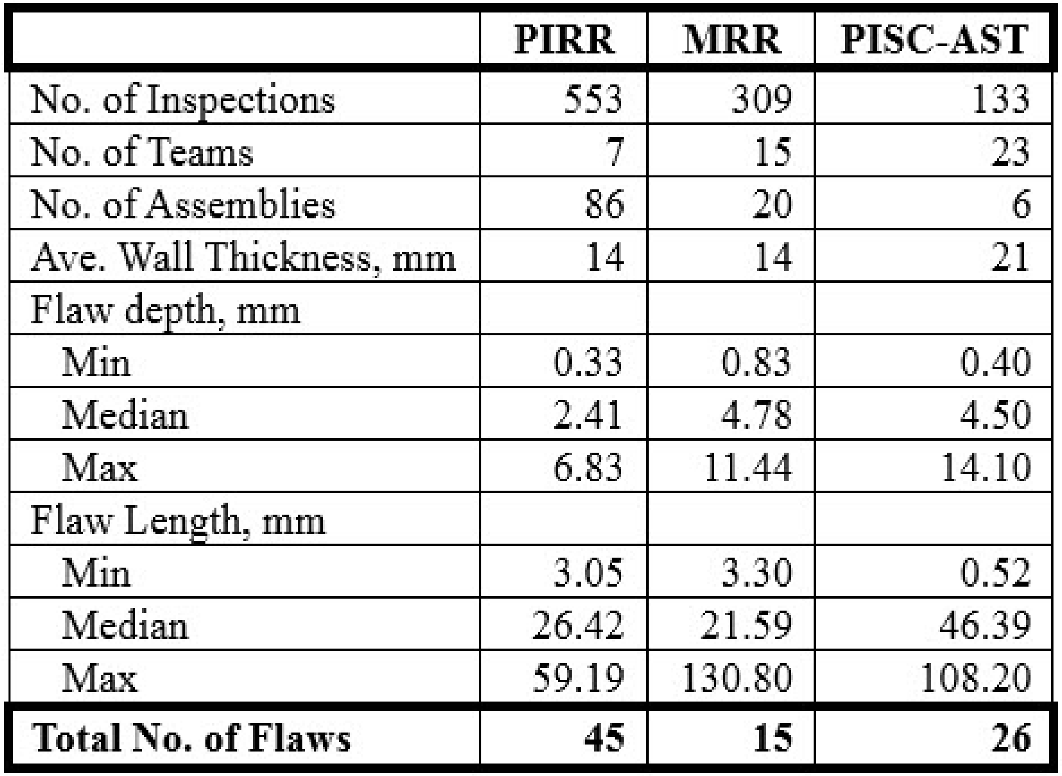
5. Tool-3, & 5 : Flaw Detection and Sizing.

NDE-UQ Approach No. 1
using
Tool-3 (LLNQ - Regression)



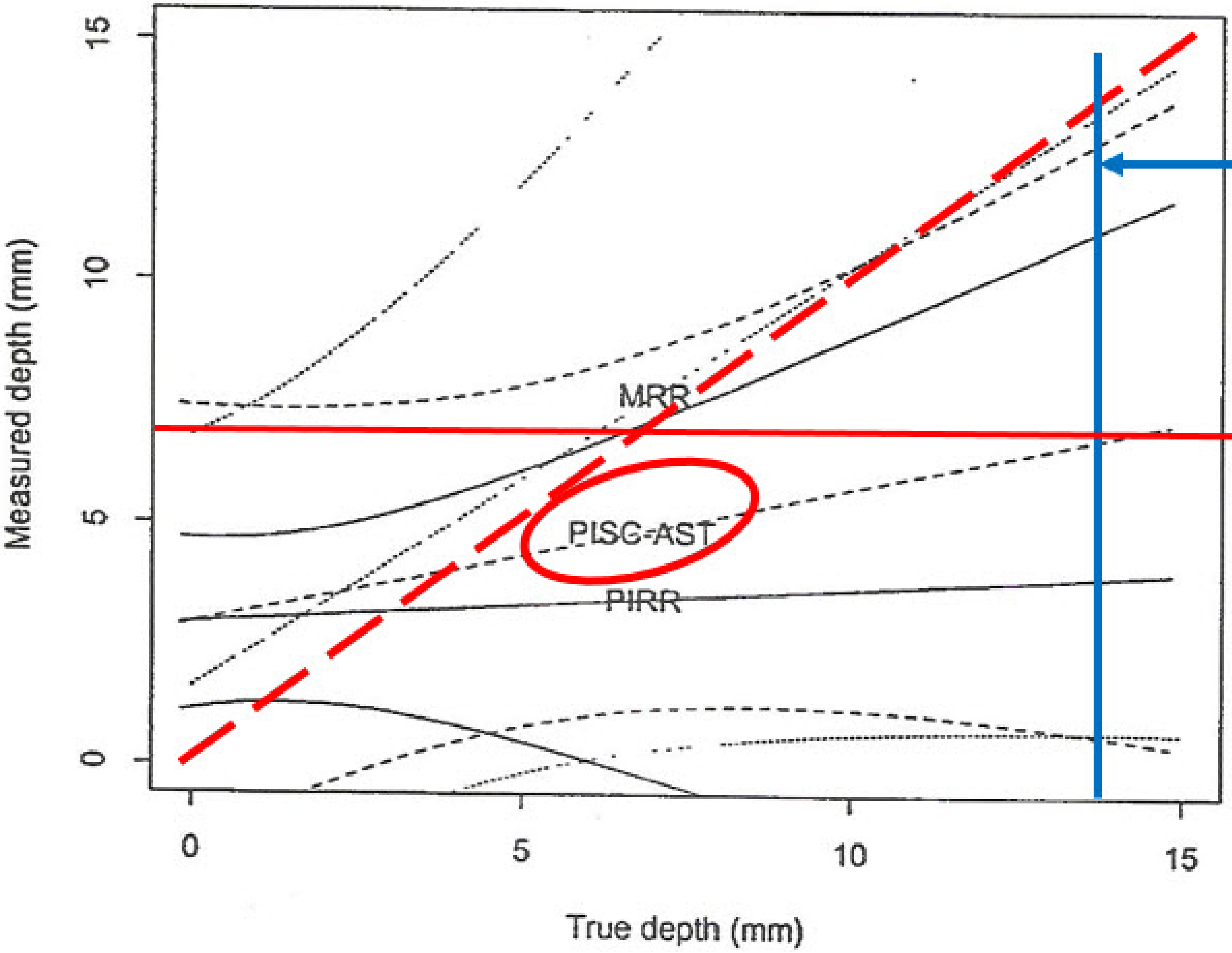
Flaw to be examined for sizing using a design of experiments.

Table 2. Summary of First 3 NDE databases reported in [17].



	PIRR	MRR	PISC-AST
No. of Inspections	553	309	133
No. of Teams	7	15	23
No. of Assemblies	86	20	6
Ave. Wall Thickness, mm	14	14	21
Flaw depth, mm			
Min	0.33	0.83	0.40
Median	2.41	4.78	4.50
Max	6.83	11.44	14.10
Flaw Length, mm			
Min	3.05	3.30	0.52
Median	26.42	21.59	46.39
Max	59.19	130.80	108.20
Total No. of Flaws	**45**	**15**	**26**

5. Tool-3, & 5 : Flaw Detection and Sizing.



13.8 mm
True Depth
(mean)

6.8 mm
Measured
Depth (mean)

Percent Error =
100*(-7.0)/ 13.8
= - 51 %

Uncertainty
Is undefined.

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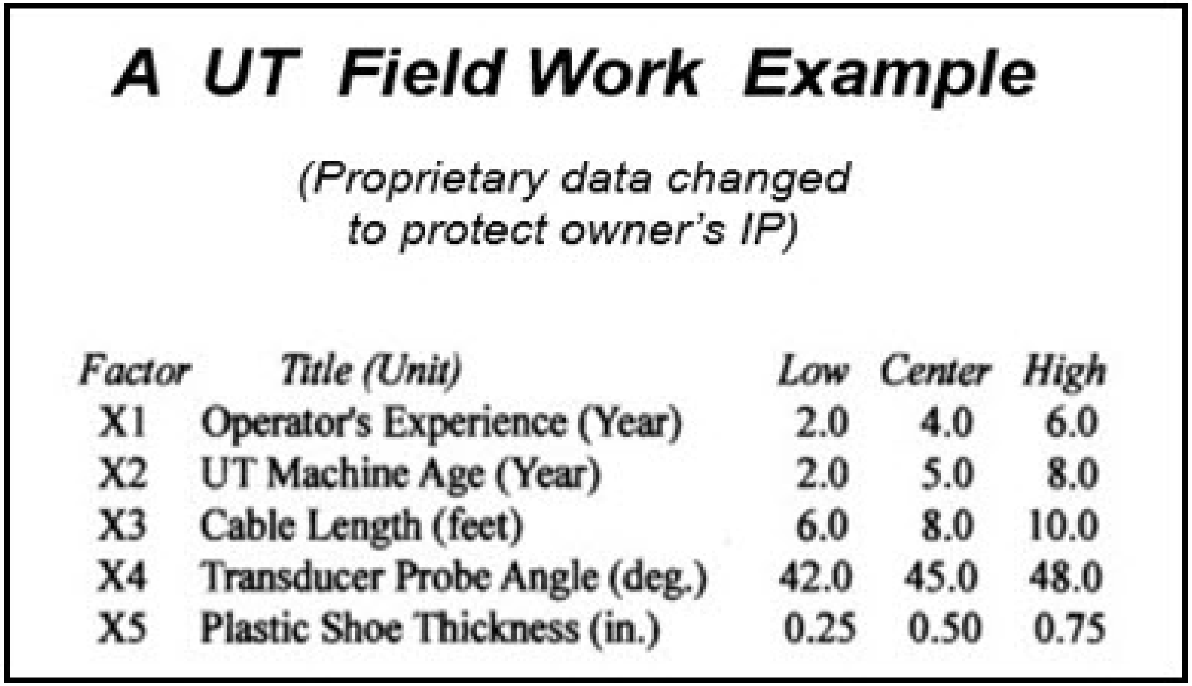
5. Tool-3, & 5 : Flaw Detection and Sizing.

NDE-UQ Approach No. 2
using
Tool-5 (DEX)

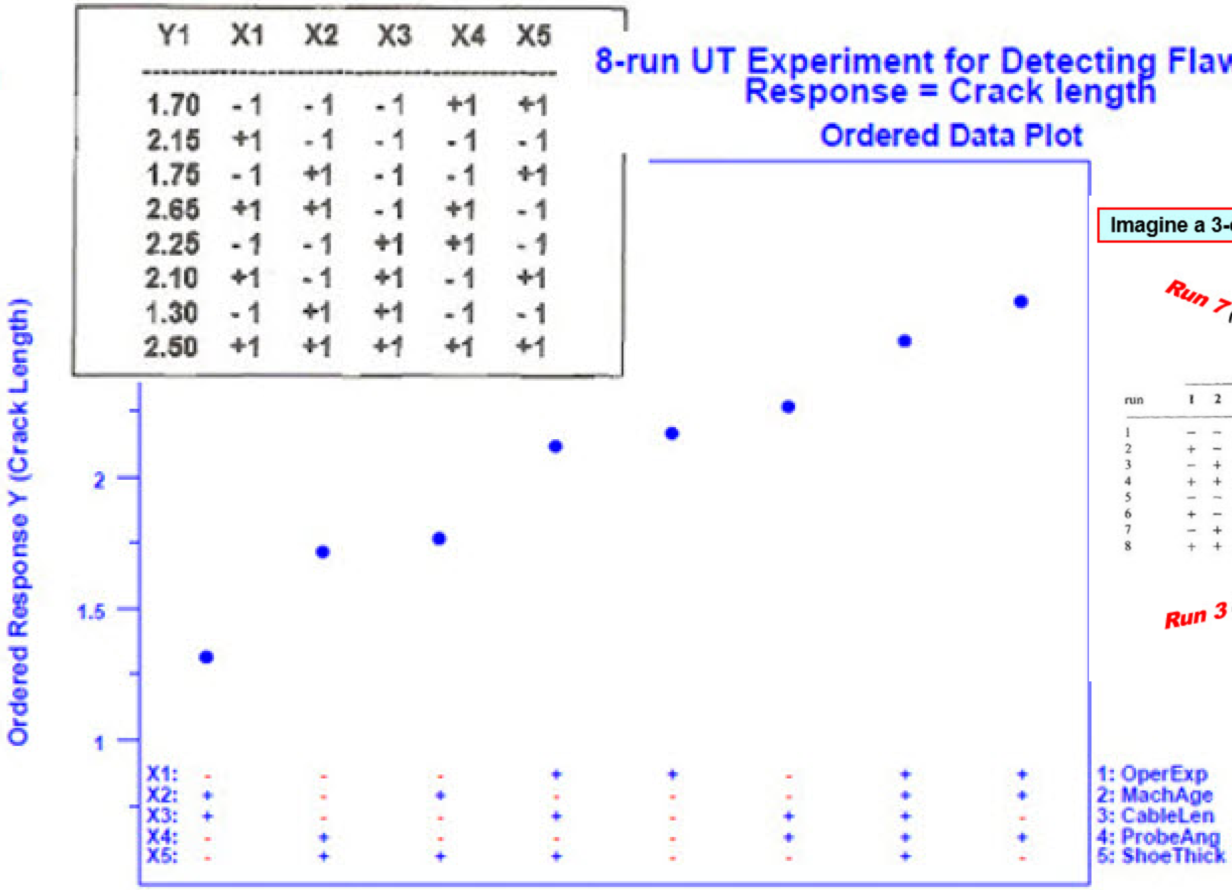


A UT Field Work Example

(Proprietary data changed to protect owner's IP)

Factor	*Title (Unit)*	*Low*	*Center*	*High*
X1	Operator's Experience (Year)	2.0	4.0	6.0
X2	UT Machine Age (Year)	2.0	5.0	8.0
X3	Cable Length (feet)	6.0	8.0	10.0
X4	Transducer Probe Angle (deg.)	42.0	45.0	48.0
X5	Plastic Shoe Thickness (in.)	0.25	0.50	0.75

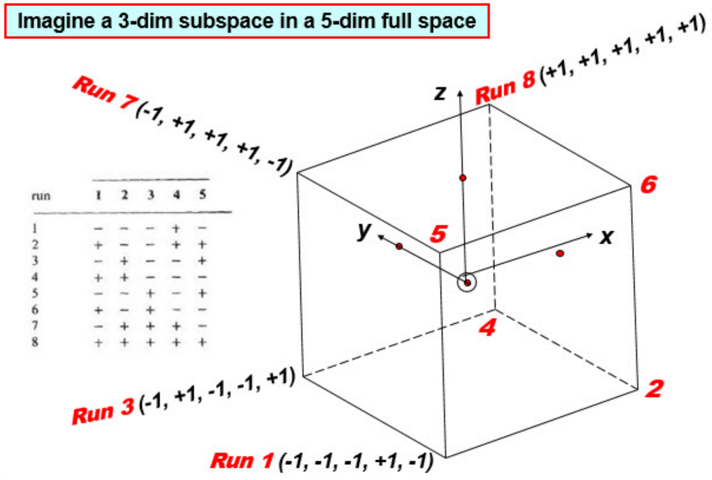
5. Tool-3, & 5 : Flaw Detection and Sizing.



Y1	X1	X2	X3	X4	X5
1.70	-1	-1	-1	+1	+1
2.15	+1	-1	-1	-1	-1
1.75	-1	+1	-1	-1	+1
2.65	+1	+1	-1	+1	-1
2.25	-1	-1	+1	+1	-1
2.10	+1	-1	+1	-1	+1
1.30	-1	+1	+1	-1	-1
2.50	+1	+1	+1	+1	+1

8-run UT Experiment for Detecting Flaw "L"
Response = Crack length
Ordered Data Plot

Imagine a 3-dim subspace in a 5-dim full space

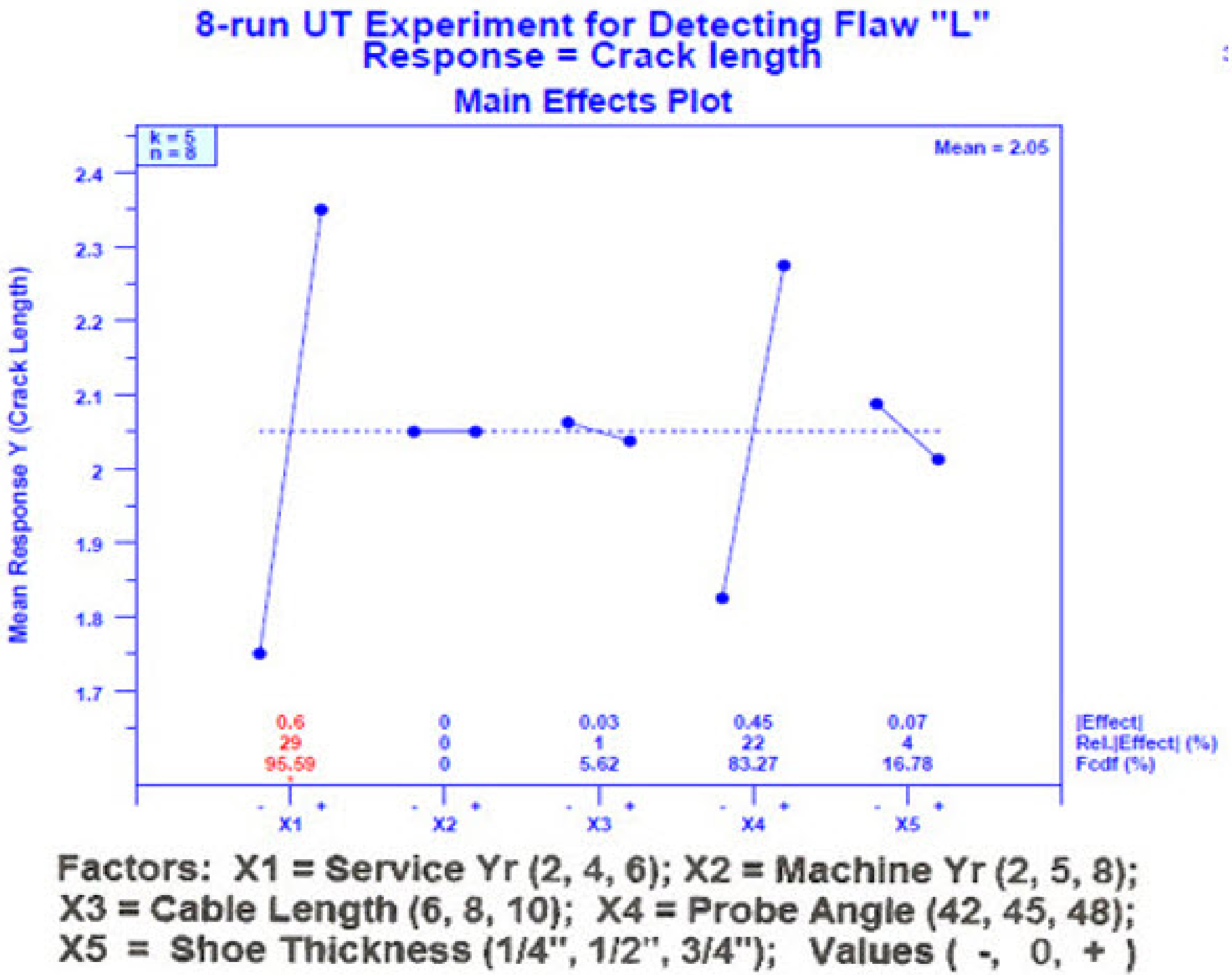


run	1	2	3	4	5
1	−	−	−	+	−
2	+	−	−	+	+
3	−	+	−	−	+
4	+	+	−	−	−
5	−	−	+	−	+
6	+	−	+	−	−
7	−	+	+	+	−
8	+	+	+	+	+

Factors: X1 = Service Yr (2, 4, 6); X2 = Machine Yr (2, 5, 8);
X3 = Cable Length (6, 8, 10); X4 = Probe Angle (42, 45, 48);
X5 = Shoe Thickness (1/4", 1/2", 3/4"); Values (-, 0, +)

2

5. Tool-3, & 5 : Flaw Detection and Sizing.



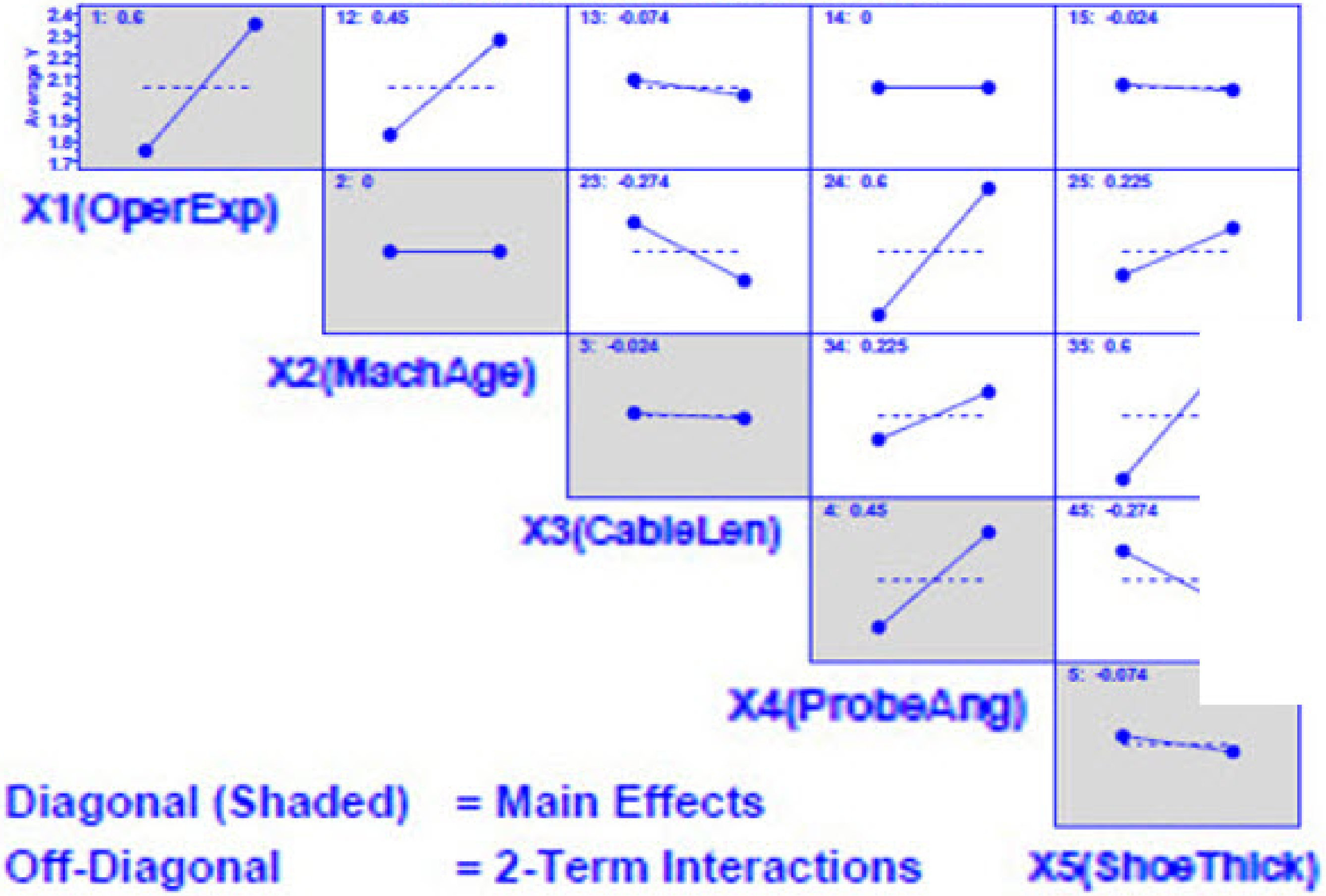
8-run UT Experiment for Detecting Flaw "L"
Response = Crack length
Main Effects Plot

Factors: X1 = Service Yr (2, 4, 6); X2 = Machine Yr (2, 5, 8);
X3 = Cable Length (6, 8, 10); X4 = Probe Angle (42, 45, 48);
X5 = Shoe Thickness (1/4", 1/2", 3/4"); Values (-, 0, +)

5. Tool-3, & 5 : Flaw Detection and Sizing.

8-run UT Experiment for Detecting Flaw "L"
Response = Crack length

Interaction Effects Matrix



Diagonal (Shaded) = Main Effects

Off-Diagonal = 2-Term Interactions

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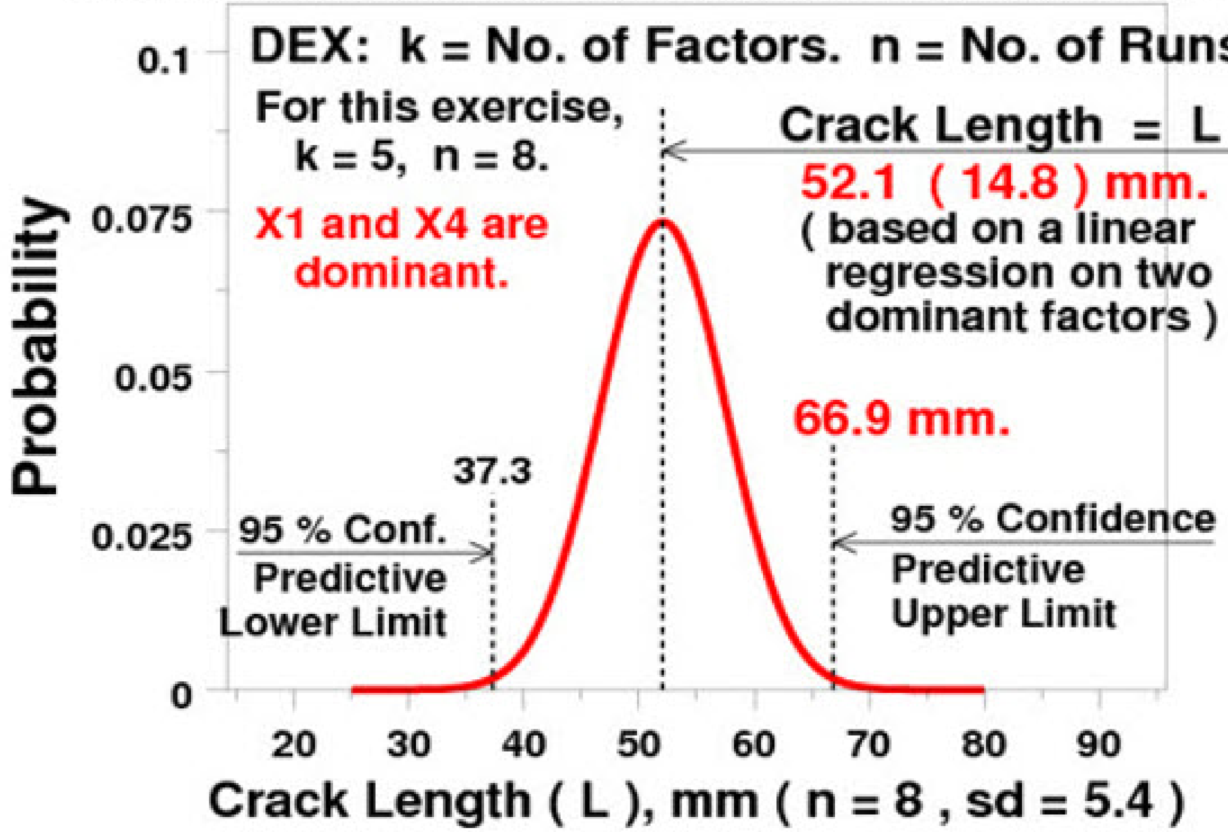
5. Tool-3, & 5 : Flaw Detection and Sizing.

Uncertainty Quantification of UT Measurement of Length of a Subsurface Crack
95 % Confidence Predictive Limits based on a Design of Experiments (DEX) Exercise



Factors: X1 = Service Yr (2, 4, 6); X2 = Machine Yr (2, 5, 8);
X3 = Cable Length (6, 8, 10); X4 = Probe Angle (42, 45, 48);
X5 = Shoe Thickness (1/4", 1/2", 3/4"); Values (-, 0, +)

No. of slides	*Subtotal*
9	*9*

6. UQ Tool-1, and 2 :

Design of an

Aircraft Window

Ref.: Fuller, Jr., E. R., Freiman, S. W., Quinn, J. B., Quinn, G. D., and Carter, W. Craig, "Fracture mechanics approach to the design of glass aircraft windows: a case study," *Proc. Conf., SPIE - The International Society for Optical Engineering*, 26-28 July 1994, San Diego, CA, Vol. 2286, pp. 419-430 (1994)

A simplified version of Equation (18) in the reference mentioned above appears on the next slide:

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6. Tool-1, 2 : Design of an Aircraft Window.

Expression for Time-to-Failure (t_f) :

$$t_f = \frac{\lambda}{N'+1}\left(\frac{S}{S_v}\right)^{N'-2}\sigma^{-N'}$$

- S is the initial strength
- S_v is the strength of an indented reference set of specimens
- **σ** is the tensile stress in the component
- **λ** and N' are constants from environmentally enhanced crack growth

#Unit of $\mathbf{t_f}$ *is* ***s****, unit of* ***S****,* $\mathbf{S_v}$*,* σ *is* ***MPa****, unit of* λ *is* $\mathbf{(MPa)^{N'}}$ *** ***s****, and* ***N'*** *is dimentionless.*

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6. Tool-1, 2 : Design of an Aircraft Window.

Use Propagation of Errors to Combine Uncertainties in the 5 Parameters

$$s.d.(t_f) = t_f \sqrt{\frac{\mathrm{var}(\lambda)}{\lambda^2} + \frac{\mathrm{var}(N')}{(N'+1)^2} + \boxed{(N'-2)^2 * \left[\frac{\mathrm{var}(S)}{S^2} + \frac{\mathrm{var}(S_v)}{(S_v)^2}\right]} + (N')^2 * \frac{\mathrm{var}(\sigma)}{\sigma^2}} \;.$$

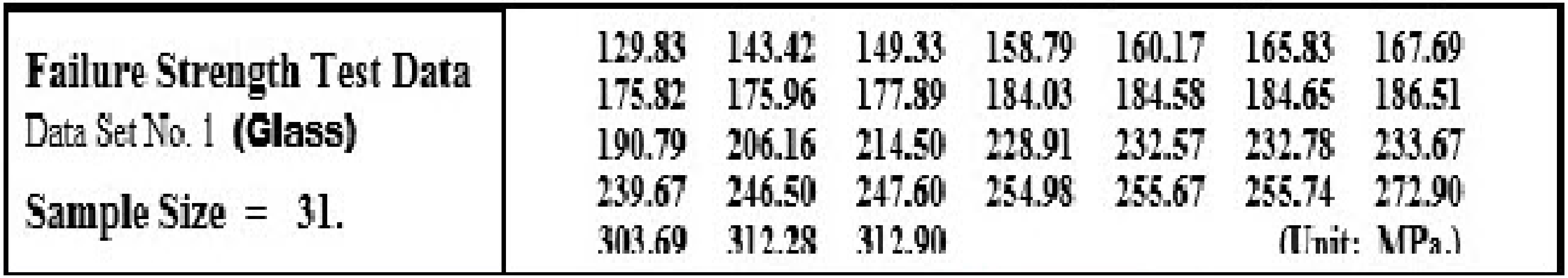
Legend:

s.d. = standard deviation.

var = variance = (s.d.)^2.

*Ref.: **Fong, J. T., Marcal, P. V., Heckert, N. A., Filliben, J. J., and Freiman, S. W.,** "Confidence Interval Estimation for Location Parameter of a 3-Parameter Weibull Distribution," submitted to **NIST Journal of Research**, Dec. 2010. Contact fong@nist.gov for advance copy of an updated version of the manuscript.*

6. Tool-1, 2 : Design of an Aircraft Window.

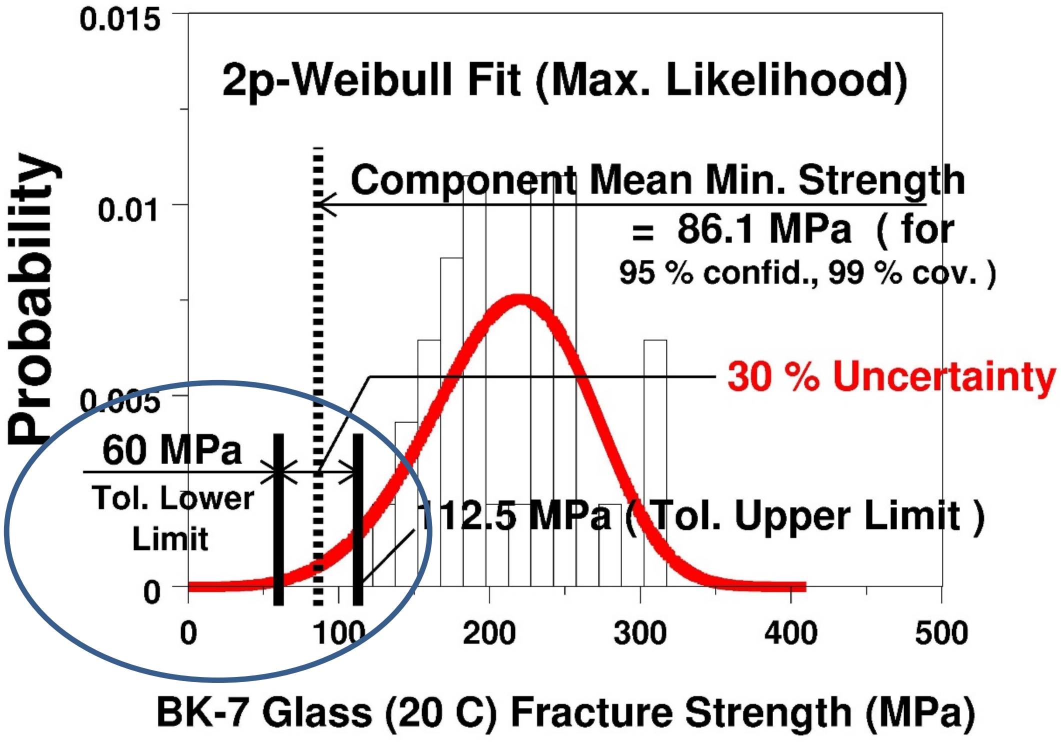


Failure Strength Test Data
Data Set No. 1 **(Glass)**

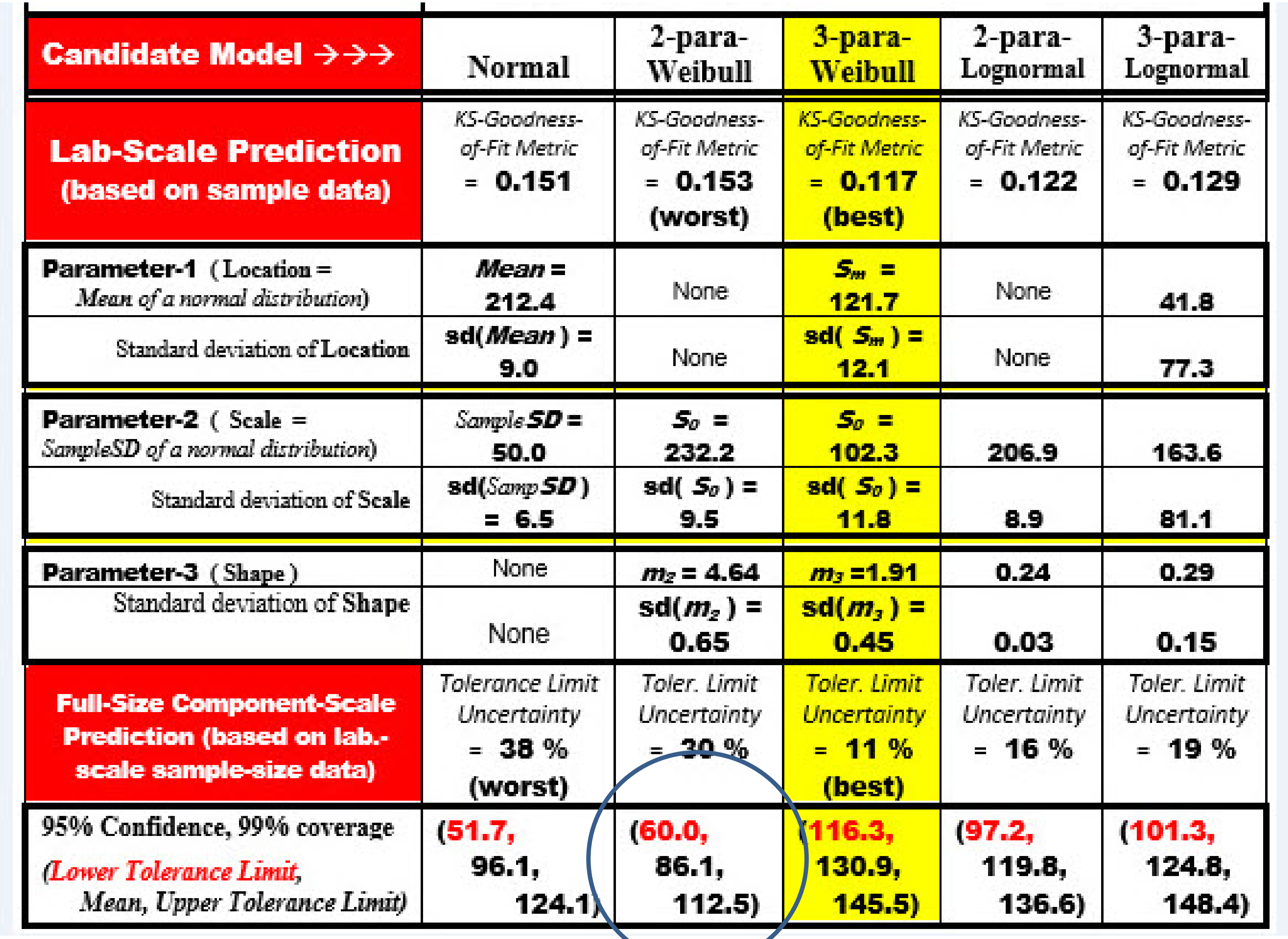
Sample Size = 31.

129.83	143.42	149.33	158.79	160.17	165.83	167.69
175.82	175.96	177.89	184.03	184.58	184.65	186.51
190.79	206.16	214.50	228.91	232.57	232.78	233.67
239.67	246.50	247.60	254.98	255.67	255.74	272.90
303.69	312.28	312.90				

(Unit: MPa.)

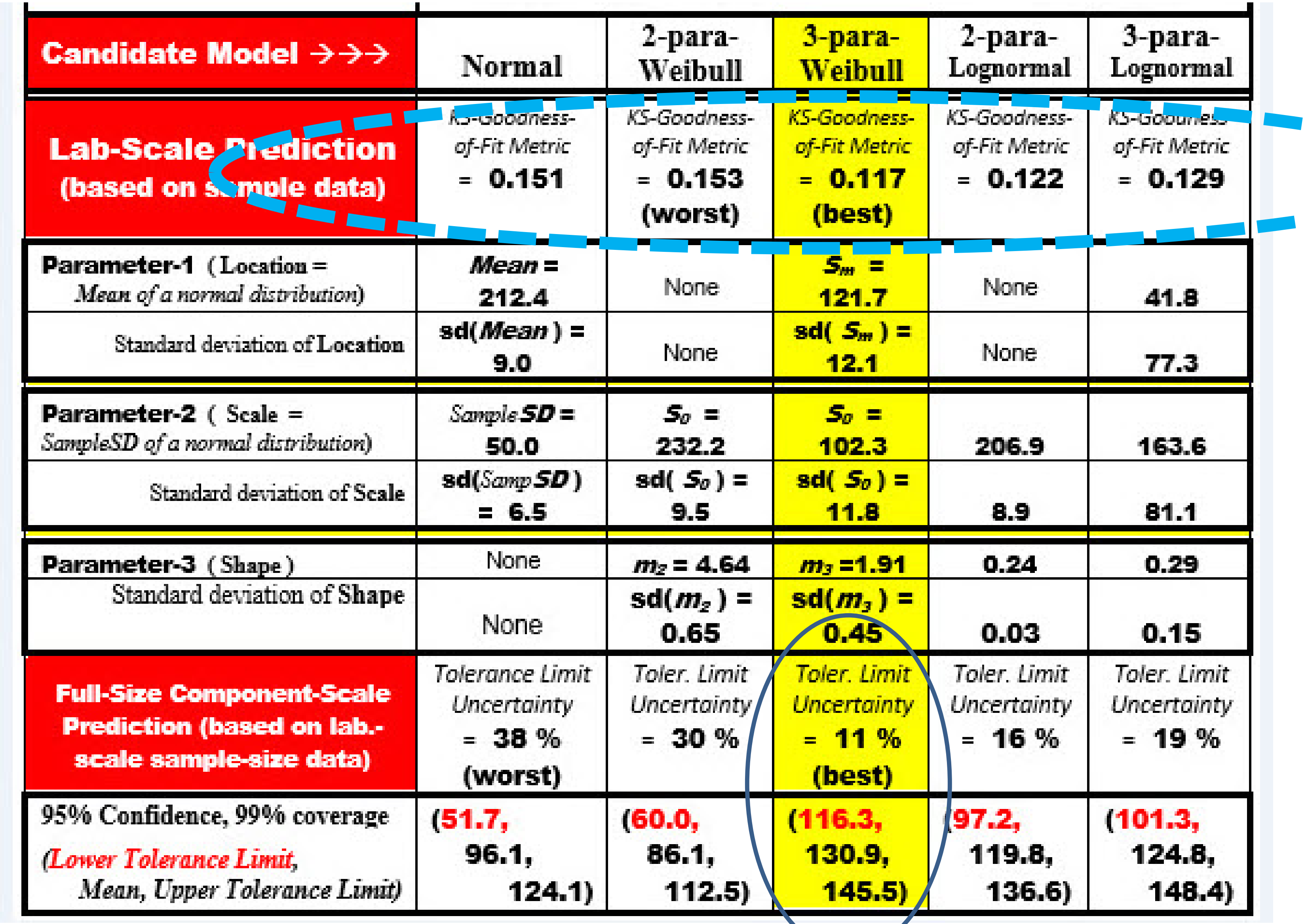


6. Tool-1, 2 : Design of an Aircraft Window.



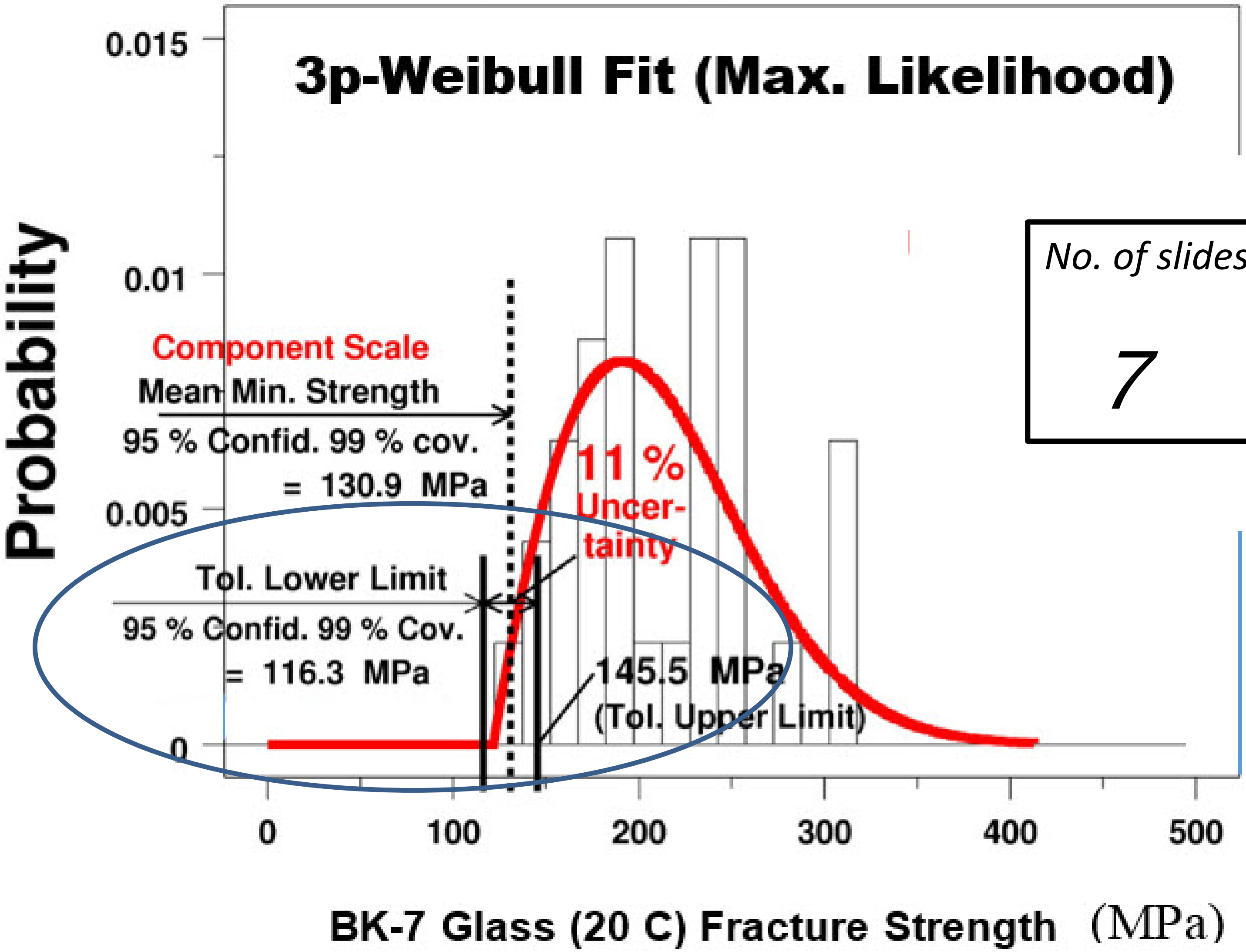
Candidate Model →→→	**Normal**	**2-para-Weibull**	**3-para-Weibull**	**2-para-Lognormal**	**3-para-Lognormal**
Lab-Scale Prediction (based on sample data)	*KS-Goodness-of-Fit Metric* = **0.151**	*KS-Goodness-of-Fit Metric* = **0.153** **(worst)**	*KS-Goodness-of-Fit Metric* = **0.117** **(best)**	*KS-Goodness-of-Fit Metric* = **0.122**	*KS-Goodness-of-Fit Metric* = **0.129**
Parameter-1 (Location = *Mean of a normal distribution*)	***Mean*** **= 212.4**	None	**S_m = 121.7**	None	**41.8**
Standard deviation of Location	**sd(*Mean*) = 9.0**	None	**sd(S_m) = 12.1**	None	**77.3**
Parameter-2 (Scale = *SampleSD of a normal distribution*)	*Sample* ***SD*** **= 50.0**	**S_0 = 232.2**	**S_0 = 102.3**	**206.9**	**163.6**
Standard deviation of Scale	**sd(*Samp* *SD*) = 6.5**	**sd(S_0) = 9.5**	**sd(S_0) = 11.8**	**8.9**	**81.1**
Parameter-3 (Shape)	None	**m_2 = 4.64**	**m_3 =1.91**	**0.24**	**0.29**
Standard deviation of Shape	None	**sd(m_2) = 0.65**	**sd(m_3) = 0.45**	**0.03**	**0.15**
Full-Size Component-Scale Prediction (based on lab.-scale sample-size data)	*Tolerance Limit Uncertainty* = **38 %** **(worst)**	*Toler. Limit Uncertainty* = **30 %**	*Toler. Limit Uncertainty* = **11 %** **(best)**	*Toler. Limit Uncertainty* = **16 %**	*Toler. Limit Uncertainty* = **19 %**
95% Confidence, 99% coverage (*Lower Tolerance Limit*, *Mean, Upper Tolerance Limit*)	**(51.7, 96.1, 124.1)**	**(60.0, 86.1, 112.5)**	**(116.3, 130.9, 145.5)**	**(97.2, 119.8, 136.6)**	**(101.3, 124.8, 148.4)**

6. Tool-1, 2 : Design of an Aircraft Window.



Candidate Model →>>	Normal	2-para-Weibull	3-para-Weibull	2-para-Lognormal	3-para-Lognormal
Lab-Scale Prediction (based on sample data)	*KS-Goodness-of-Fit Metric* = **0.151**	*KS-Goodness-of-Fit Metric* = **0.153** **(worst)**	*KS-Goodness-of-Fit Metric* = **0.117** **(best)**	*KS-Goodness-of-Fit Metric* = **0.122**	*KS-Goodness-of-Fit Metric* = **0.129**
Parameter-1 (Location = *Mean of a normal distribution*)	***Mean*** = **212.4**	None	S_m = **121.7**	None	**41.8**
Standard deviation of Location	**sd(*Mean*) = 9.0**	None	**sd(** S_m **) = 12.1**	None	**77.3**
Parameter-2 (Scale = *SampleSD of a normal distribution*)	*Sample* ***SD*** = **50.0**	S_0 = **232.2**	S_0 = **102.3**	**206.9**	**163.6**
Standard deviation of Scale	**sd(*Samp SD*) = 6.5**	**sd(** S_0 **) = 9.5**	**sd(** S_0 **) = 11.8**	**8.9**	**81.1**
Parameter-3 (Shape)	None	m_2 **= 4.64**	m_3 **=1.91**	**0.24**	**0.29**
Standard deviation of Shape	None	**sd(** m_2 **) = 0.65**	**sd(** m_3 **) = 0.45**	**0.03**	**0.15**
Full-Size Component-Scale Prediction (based on lab.-scale sample-size data)	*Tolerance Limit Uncertainty* = **38 %** **(worst)**	*Toler. Limit Uncertainty* = **30 %**	*Toler. Limit Uncertainty* = **11 %** **(best)**	*Toler. Limit Uncertainty* = **16 %**	*Toler. Limit Uncertainty* = **19 %**
95% Confidence, 99% coverage (*Lower Tolerance Limit*, *Mean, Upper Tolerance Limit*)	**(51.7, 96.1, 124.1)**	**(60.0, 86.1, 112.5)**	**(116.3, 130.9, 145.5)**	**(97.2, 119.8, 136.6)**	**(101.3, 124.8, 148.4)**

6. Tool-1, 2 : Design of an Aircraft Window.



No. of slides	*Subtotal*
7	*16*

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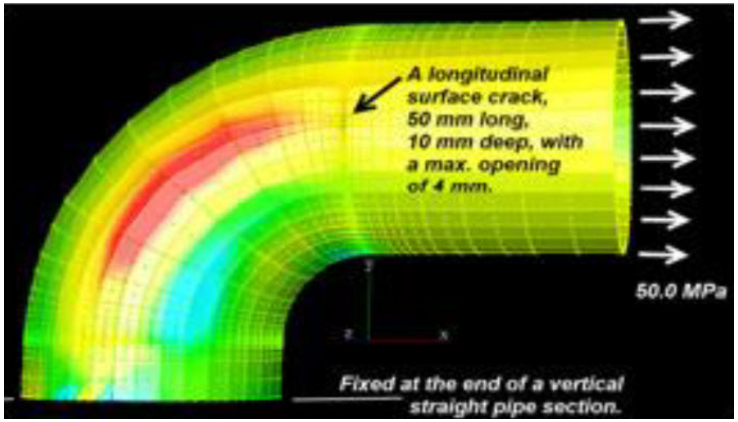
7. UQ Tool-6 Application:

Maintenance

Decision Making.

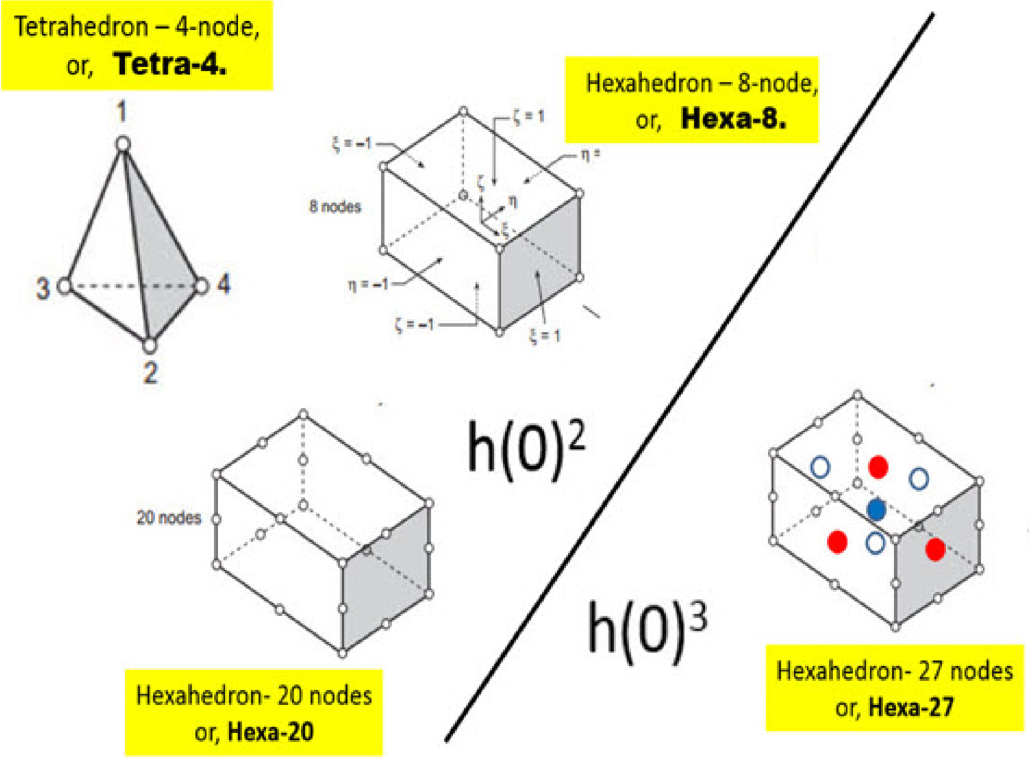
7. Tool-6 Application: Maintenance Decision Making.



A longitudinal surface crack, 50 mm long, 10 mm deep, with a max. opening of 4 mm.

50.0 MPa

Fixed at the end of a vertical straight pipe section.



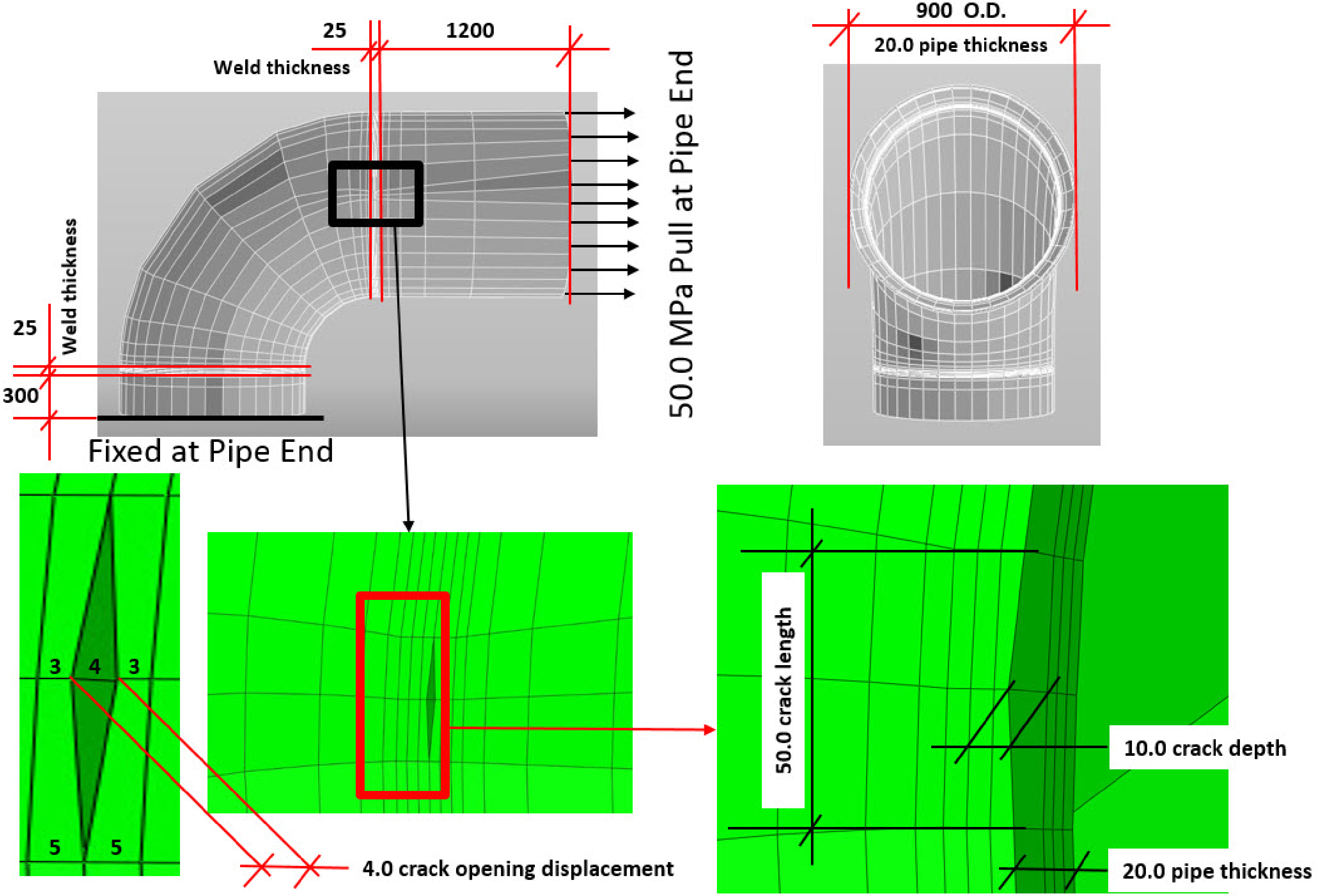
Tetrahedron – 4-node, or, **Tetra-4.**

Hexahedron – 8-node, or, **Hexa-8.**

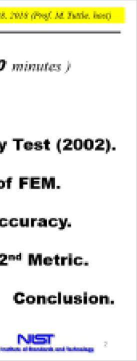
Hexahedron- 20 nodes or, **Hexa-20**

Hexahedron- 27 nodes or, **Hexa-27**

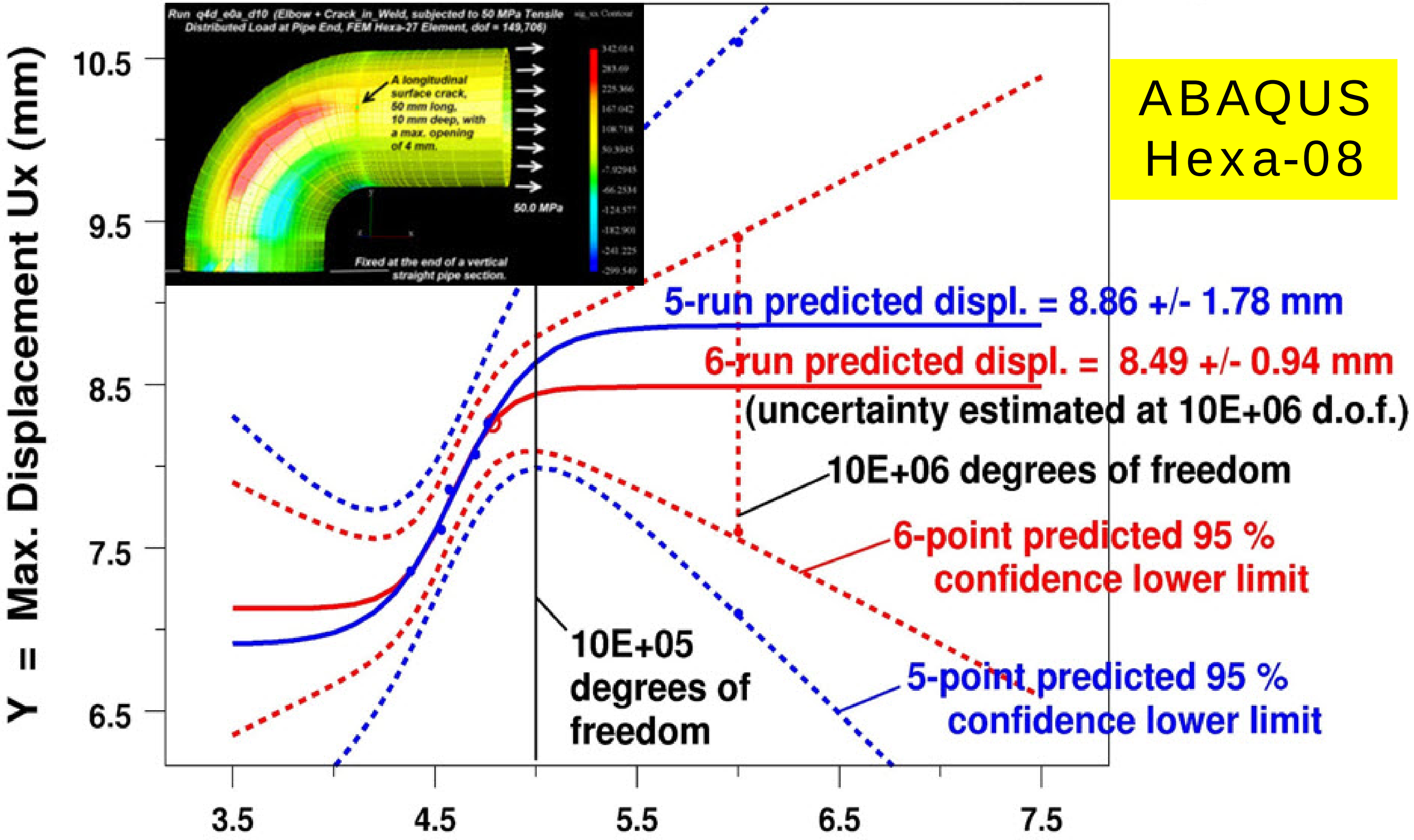
7. Tool-6 Application: Maintenance Decision Making.



7. Tool-6 Application: Maintenance Decision Making.


8, 2018 (Prof. M. Tuttle, host)
0 minutes)
y Test (2002).
of FEM.
ccuracy.
2nd Metric.
Conclusion.

4-para. Logistic : Y = y1 - L*(exp(-k*(xx-x0)) / (1 + exp(-k*(xx-X0)))) 2
where xx = Log_10(X) (Fong-Filliben-Heckert-Marcal-Rainsberger-Ma, 2015)



LOG_10 (X) where X = degrees of freedom (d.o.f.) of
ABAQUS Solutions of Elbow-Crack Problem with 5 runs of Hex-08 (blue dots) and extra 6th run (red circle).

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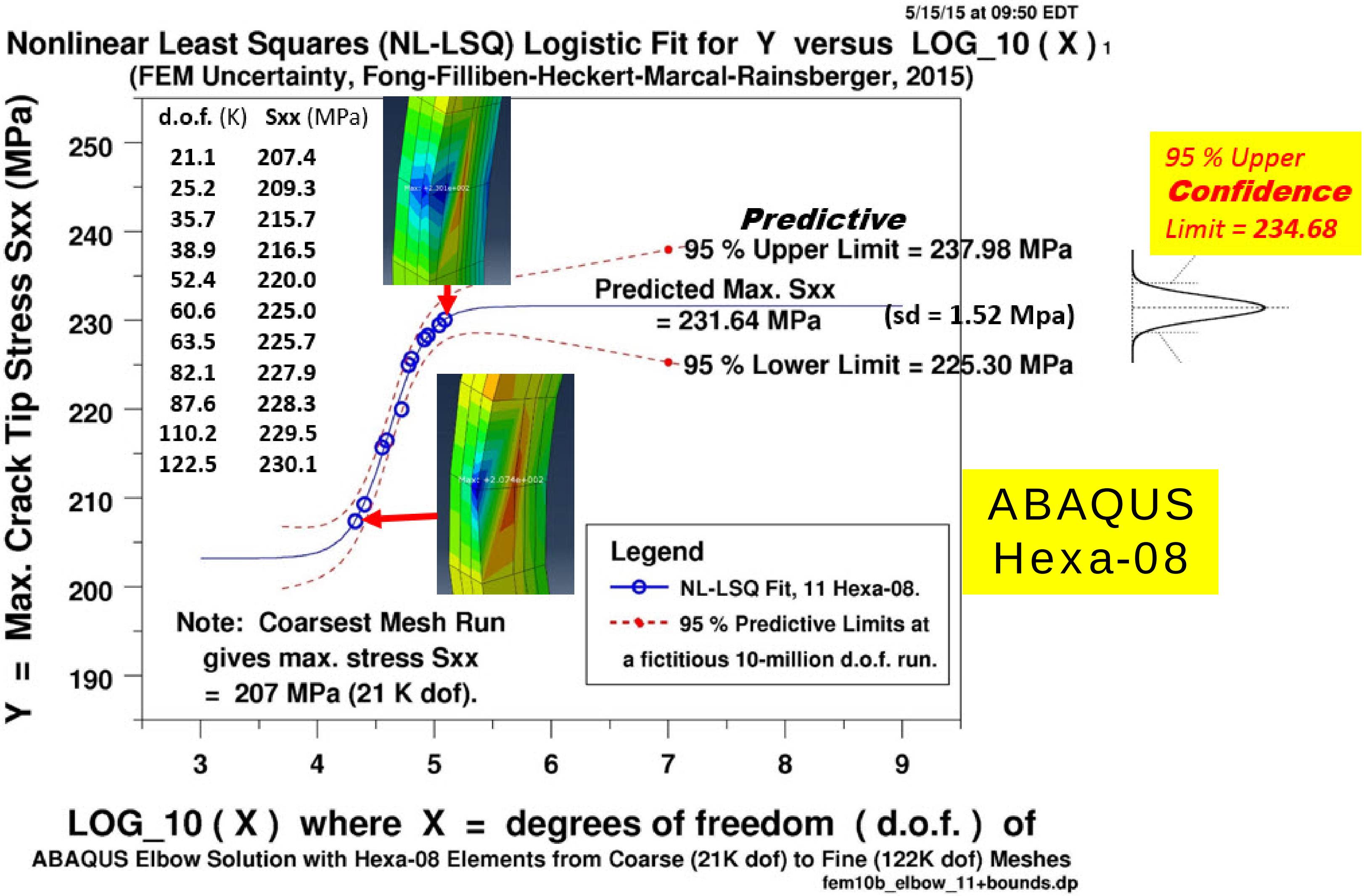
7. Tool-6 Application: Maintenance Decision Making.



5/15/15 at 09:50 EDT

Nonlinear Least Squares (NL-LSQ) Logistic Fit for Y versus LOG_10 (X) 1

(FEM Uncertainty, Fong-Filliben-Heckert-Marcal-Rainsberger, 2015)

d.o.f. (K)	Sxx (MPa)
21.1	207.4
25.2	209.3
35.7	215.7
38.9	216.5
52.4	220.0
60.6	225.0
63.5	225.7
82.1	227.9
87.6	228.3
110.2	229.5
122.5	230.1

Predictive
95 % Upper Limit = 237.98 MPa
Predicted Max. Sxx
= 231.64 MPa
(sd = 1.52 Mpa)
95 % Lower Limit = 225.30 MPa

Note: Coarsest Mesh Run
gives max. stress Sxx
= 207 MPa (21 K dof).

Legend
NL-LSQ Fit, 11 Hexa-08.
95 % Predictive Limits at
a fictitious 10-million d.o.f. run.

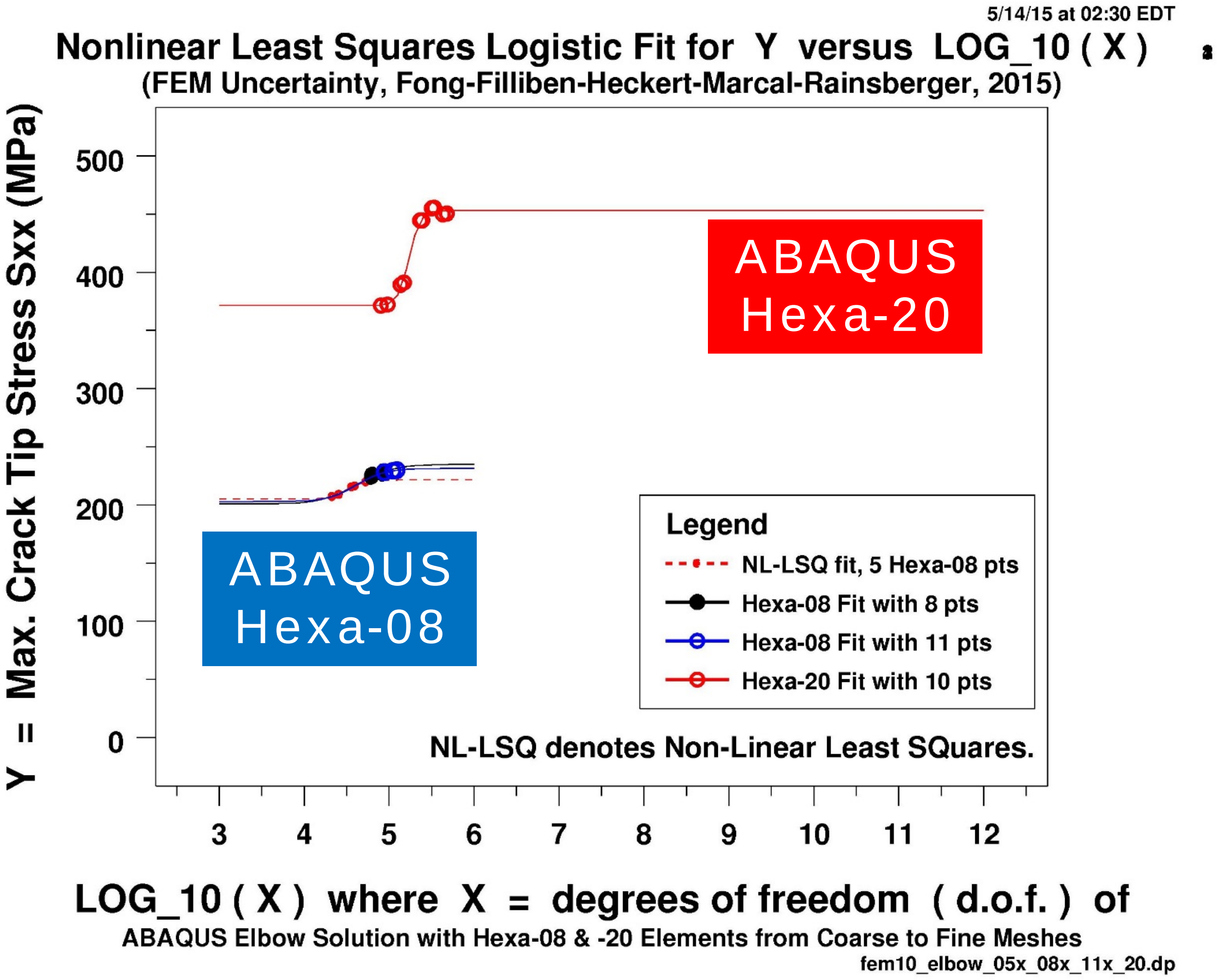
95 % Upper ***Confidence*** *Limit = 234.68*

ABAQUS
Hexa-08

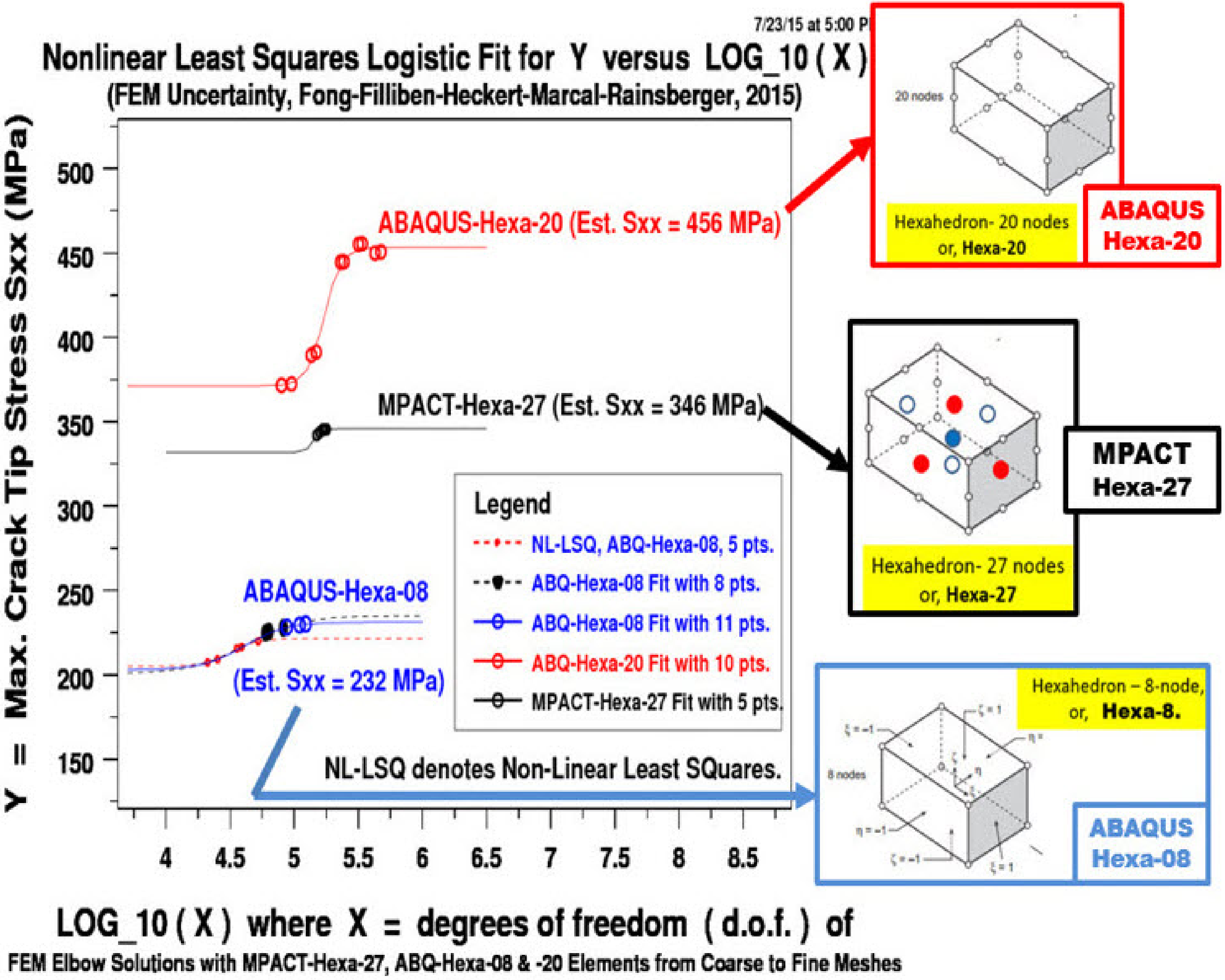
LOG_10 (X) where X = degrees of freedom (d.o.f.) of
ABAQUS Elbow Solution with Hexa-08 Elements from Coarse (21K dof) to Fine (122K dof) Meshes

fem10b_elbow_11+bounds.dp

7. Tool-6 Application: Maintenance Decision Making.



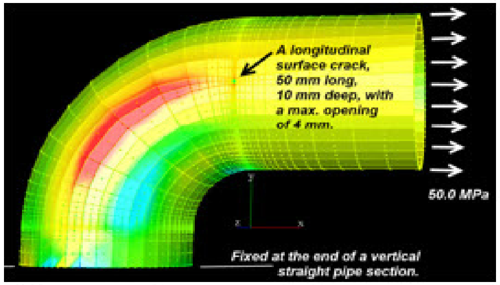
7. Tool-6 Application: Maintenance Decision Making.



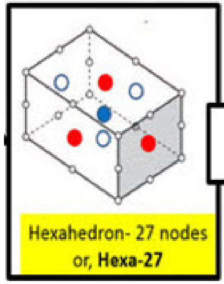
No. of slides	*Subtotal*
8	*24*

A 900-mm (36-in) o.d. Pipe 90-deg. Elbow with a surface crack in one of its two welds

Est. Max. Crack Tip Stress ***Sxx (MPa)*** *at*
1 billion *(10^9)* ***degrees of freedom***
using a Nonlinear Least Squares Logistic Fit of 5 or more FEM solutions of the same mesh design at increasing mesh densities



ABAQUS Hexa-20


Hexahedron- 27 nodes or, Hexa-27

MPACT Hexa-27

ABAQUS Hexa-08

FEM Code-Element Type *No. of Runs* (Best Estimated Solution)	95 % Lower Limit at 10^9 d.o.f. (MPa)	Predicted Max. Crack Tip Stress at 10^9 d.o.f. (MPa)	95 % Upper Limit at 10^9 d.o.f. (MPa)		Stand. Dev. (S.D.) at 10^9 d.o.f. (MPa)	Coeff. of Variation (C.V.) at 10^9 d.o.f. (%)		Ranking of Solutions by C.V. (least being the best)
ABQ-Hex20 ***7 runs*** (455.20)	407.32	**457.96**	508.60		**19.70**	**4.30 %**		**6**
ABQ-Hex20 ***9 runs*** (455.50)	413.80	**454.23**	494.67		**17.10**	**3.76 %**		**5**
ABQ-Hex20 ***10 Runs*** (455.50)	418.74	**453.17**	487.61		**14.93**	**3.29 %**		**4**
MPACT-Hex27 ***5 runs*** (345.48)	345.10	**345.47**	345.85		**0.12**	**0.03 %** ***(lowest)***		**1**
ABQ-Hex08 ***5 runs*** (220.00)	203.02	**246.05**	289.09		**13.52**	**5.49 %**		**7**
ABQ-Hex08 ***9 runs*** (228.30)	215.78	**233.37**	250.96		**7.44**	**3.19 %**		**3**
ABQ-Hex08 ***11 runs*** (230.10)	220.56	**231.69**	242.82		**4.92**	**2.12 %**		**2**